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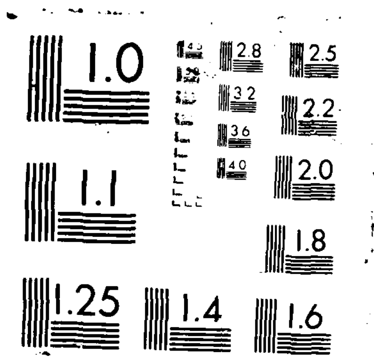
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AN IMPROVED METHOD FOR CALCULATING
POWER DENSITY IN THE FRESNEL REGION OF
CIRCULAR PARABOLIC REFLECTOR ANTENNAS

THESIS

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AN IMPROVED METHOD FOR CALCULATING POWER DENSITY IN THE FRESNEL REGION OF CIRCULAR PARABOLIC REFLECTOR ANTENNAS

THESIS

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Preface

The goal of this thesis was to develop a better method for Air Force use in evaluating Fresnel region power density hazards associated with paraboloidal antennas. I wanted to not only improve on the aperture illumination model, but improve on the entire procedure for calculating the power density figures. The resulting computer program, POWERDEN, utilizes the one-parameter circular aperture distribution developed by R. C. Hansen, and I want to publicly acknowledge his work here. I've attempted to make the program as user-friendly as possible, although it is designed for use by those people already familiar with paraboloidal radiation and the related issues of electromagnetic compatibility. Besides absolute power density and distance figures, the output in both the axial and off-axis modes is also given in relative terms so that graphs based on the data can be more easily constructed. I hope that my efforts have resulted in a useful product which the Air Force can actually use to help protect its people and equipment from hazardous radiation.

I'd like to thank Mr. Warren L. Keller and the people at 1842 EEG/EEITE, Scott AFB for sponsoring the project and for their hospitality during my visit. I'd also like to thank my thesis advisor, Dr. Andrew Terzuoli, and the other

members of my committee, Capt. Randy Jost and Dr. Alan Lair, for there help and suggestions along the way.

Finally, I'd like to thank my wife, Linda, and my sons, Adam and Joshua, for their support during the writing of this paper and during my entire time at AFIT. The seemingly endless hours of work and study over the past 21 months were possible only because of their love and patience.

Johnnie E. Mize

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Abstract

A computer program is presented which calculates power density in the Fresnel region of circular parabolic reflector antennas. The aperture illumination model is the one-parameter circular distribution developed by Hansen. The program is applicable to the analysis of electrically large, center-fed (or Cassegrain) paraboloids with linearly polarized feeds.

The scalar Kirchoff diffraction integral is solved numerically by Romberg integration for points both on and perpendicular to the antenna boresight. Axial results correspond with those published by Hansen. Off-axis results cannot be directly compared to any others obtained with this illumination model, but they are consistent with what is expected in the Fresnel region where a quadratic must be added to the linear phase term of the integral expression. Graphical results are presented for uniform illumination and for cases where the first sidelobe ratio is 20, 25, 30, and 35 dB.

AN IMPROVED METHOD FOR CALCULATING POWER DENSITY IN THE FRESNEL REGION OF CIRCULAR PARABOLIC REFLECTOR ANTENNAS

I. Introduction

Overview

Circular parabolic reflector antennas (paraboloids) are used extensively by both the military and civilian sectors for microwave communication, radar tracking, and deep-space applications. Relative design simplicity coupled with high power gain and symmetric, pencil-beam radiation patterns make them well suited for these tasks. However, the same energy focusing characteristics that help make the paraboloid so useful can also result in hazardous power densities in their vicinity that are of concern because of potential biological damage such as cataracts, interference with pacemakers or other electronic equipment, and ignition of nearby explosive material (Hansen, 1976b:50; Farrar and Adams, 1980:134; Department of the Air Force, 1981: 1-1; Michaelson, 1980:40-49).

In most instances the hazardous power densities occur within the antennas' Fresnel region (radiating near field). In this region, power density calculations are more difficult than those associated with the far field where radial field components are negligible, and all rays follow nearly parallel paths to the observation point. Power density in the far field varies simply with the inverse

square of distance according to

$$S = PG/4\pi R^2 \qquad (1)$$

where

S = power density
G = far field power gain
R = far field range

Use of Eq (1) in the Fresnel region gives answers that are too large, since gain is degraded in that region (Hansen, 1976b:52). Rather than being constant, Fresnel region gain fluctuates in a sinusoidal manner due to constructive and destructive interference among aperture field components with different amplitudes and phases (Jull, 1981:46).

The method used now by the Air Force for determining these hazardous Fresnel region power densities is outlined in Chapter 6 of Technical Order 31Z-10-4 and assumes an aperture illumination that is rotationally symmetric in amplitude, uniform in phase, and linearly polarized. The model is of the form

$$G(x) = (1-x^2)^N \qquad (2)$$

where

$G(x)$ = normalized aperture distribution
x = ρ/a, normalized aperture radial variable
ρ = aperture radial variable
a = aperture radius
N = exponent controlling rate of amplitude taper

and is deficient in the following respects:

1. It results in zero amplitude at the aperture edge. This implies either an infinite aperture or the perfect

interception of all energy radiated by the feed horn. Neither condition occurs in practice (Sciambi, 1965:79; Rabello and Nobili, 1973:677; Hansen, 1976b:50); instead, the amplitude tapers off to some minimum, non-zero value.

2. The taper control exponent is restricted to the integer values of 0, 1, 2, or 3. There is no physical reason why this parameter should be restricted to integer values and, for highly tapered distributions, it can exceed the value of three (Skolnik, 1970:9-20; Silver, 1949:194; Farrar and Adams, 1980:135).

3. The procedure for estimating the taper control exponent is tied to the beamwidth. This relationship is ambiguous, and the estimate is checked by calculating antenna efficiency using a gain factor corresponding to each of the four allowable integer values of the exponent. Succesive values of the gain factor are plugged into the efficiency equation until the value falls anywhere within the range 0.5 to 0.9. This procedure is imprecise, and it is possible that none of the four gain factors will result in an acceptable efficiency figure. Beamwidth is not a good indication of any particular illumination because it is related to antenna size as well as the field distribution across the aperture, i.e. even if the illumination remains the same, the beamwidth can be halved simply by doubling the antenna diameter. The rate of amplitude taper is actually related to the level of the first sidelobe of the far field pattern (Farrar and Adams, 1980:134; Skolnik, 1970:9-25).

Problem Statement

The goal of this thesis is the development of a better method for Air Force use in calculating the Fresnel region power densities associated with paraboloidal antennas. The method will utilize an aperture illumination model that corrects the three deficiencies discussed above and will be incorporated into a computer program. It will directly calculate axial power density along the antenna boresight as well as power density off-axis so that a radiation hazard radius based on existing standards can be determined.

Assumptions

1. The paraboloid is electrically large. This condition is almost always met when using practical antennas at microwave frequencies and explains why paraboloidal design is based on high frequency, optical methods such as geometric optics and ray tracing. Methods such as GTD which account for diffraction from the rim are required only for extreme accuracy or for wide angles encompassing the minor lobes (Balanis, 1982:612-613). The method developed in this report is based on a minimum aperture diameter of 10λ.

2. The paraboloid is focused at infinity. Far field operation is usually the goal so that this requirement is met in the majority of cases. Reflector antennas are sometimes defocused for the purpose of maximum energy transfer as the target approaches or comes within the antenna's near field (Kay, 1960:587).

3. The aperture illumination is rotationally symmetric in amplitude. This is an ideal condition, but designers usually do try to realize symmetric distributions (Hansen, 1964: 64; Rudge et al., 1982:42) so that the subsequent analysis will be simpler.

4. The aperture illumination is uniform in phase. This is also an ideal condition that can never be totally realized because of unavoidable imperfections in the paraboloid's construction. Although deviations from constant phase might be planned for beam steering or some other specialized purpose (Jull, 1981:26), they are usually unplanned and result in lower gain, higher sidelobes, and wider beamwidth (Skolnik, 1980:233).

5. The electric field is linearly polarized. The assumption of linear polarization along with the high frequency approximation addressed earlier allows calculation of the diffracted field along the antenna boresight without regard to the field's vector nature, i.e. it becomes a scalar diffraction problem (Silver, 1949:164). Although the cross-polarized field components do not cancel completely at points located off-axis, their effect is usually neglected since most practical paraboloids focus energy in a very narrow angular region around the axis of symmetry.

6. The effects of aperture blockage are neglected. The center feed horn (or hyperbolic subreflector in the Cassegrain case) and its related supports are unavoidable obstacles which alter the radiation pattern of paraboloidal

antennas. Although the effect of the supports has proven difficult to analyse, it has been shown by superposition of aperture illuminations that the effect of the central blockage is to decrease gain, increase the level of the odd numbered sidelobes, and decrease the level of the even numbered sidelobes (Clarke and Brown, 1980:207-209; Rudge et al., 1982:142-145).

7. The antenna's Fresnel region will be assumed to begin a few wavelengths in front of the antenna and continue out to the conventionally agreed upon beginning of the far field at a distance of $2D^2/\lambda$, where D is the paraboloid's diameter and λ is the wavelength. Out to a few wavelengths away the reactive field dominates and no approximations can be made to the field integral (Silver, 1949:170). The $2D^2/\lambda$ limit is based on the criteria that no phase difference on axis can exceed $\pi/8$ radians or 22.5 degrees (Skolnik, 1970:9-9). Although it may be inadequate for highly tapered distributions which result in extremely low sidelobe levels (Hansen, 1964:31), this far field criterion is widely accepted.

8. The reader is expected to have an understanding of basic antenna concepts such as gain, beamwidth, aperture illumination, sidelobe level, and far field radiation characteristics.

Scope

The method developed for this thesis will be applicable to standard, center-fed paraboloids or to dual reflectors of the Cassegrain type that employ conventional pyramidal or conical feed horns. It will not, in general, be applicable to systems with less common features such as offset or corrugated feed horns, since the previous assumptions of a rotationally symmetric aperture distribution and of negligible cross-polarized components along the boresight might not be valid. The resulting computer program is expected to be a useful tool for Air Force use in determining the location of hazardous Fresnel region power densities.

Method

The approach taken will be the development of a field integral equation that is valid in the Fresnel Region. The aperture illumination model will be the one-parameter circular distribution developed by Hansen and discussed further in Chapter 3. The coupling of a non-zero amplitude pedestal at the aperture edge and a symmetric amplitude taper makes it a more realistic model (Sciambi, 1965:79; Rabello and Nobili, 1973:677; Hansen, 1976b:50,52; Hansen, 1964:50,64) than the present one given by Eq (2). The computer program will perform the necessary numerical integration using the Romberg adaptive trapezoidal technique and will directly calculate power density at discreet points

both on and off-axis. Results will be compared and contrasted with the theoretical work of Hansen and others.

Development

Information on concepts relevant to paraboloids and the Fresnel region in general is given in Chapter 2 while Chapter 3 presents the results of a literature search on the subject of determining power density in the Fresnel region. Chapter 4 explains the design and use of POWERDEN, the computer program produced as a result of this thesis. Graphical results of the program's use are presented in Chapter 5 where they are analysed in relation to the findings of Hansen and others. Chapter 6 presents some final thoughts and recommendations for further study.

II. Background

Overview

This chapter attempts to summarize some basic concepts upon which the determination of Fresnel region power density is based. Paraboloidal design is discussed and the characteristics of the field within the Fresnel region are expanded upon since they may not be as familiar as those which apply to the far field. The concept of Fresnel zones is introduced because it is a convenient way to visualize the process which results in the sinusoidal variation of axial power density in the Fresnel region. The chapter concludes with a discussion of the approximations which are usually made to the field integral equation when the observation point is within the Fresnel region.

Paraboloidal Design

The principle upon which the design of paraboloids of typical dimensions is based has been summarized as follows:

> It has been shown by geometrical optics that if a beam of parallel rays is incident upon a reflector whose geometrical shape is a parabola, the radiation will converge (focus) at a spot which is known as the focal point. In the same manner, if a point source is placed at the focal point, the rays reflected by a parabolic reflector will emerge as a parallel beam. (Balanis, 1982:604)

This principle is illustrated in Figure 1 for the case of a circular parabolic reflector. In the limit of zero wavelength and ideal conditions, a spherical wave emanating

from the focal point is tranformed into a plane wave after reflection. The reverse is true for the receive mode of

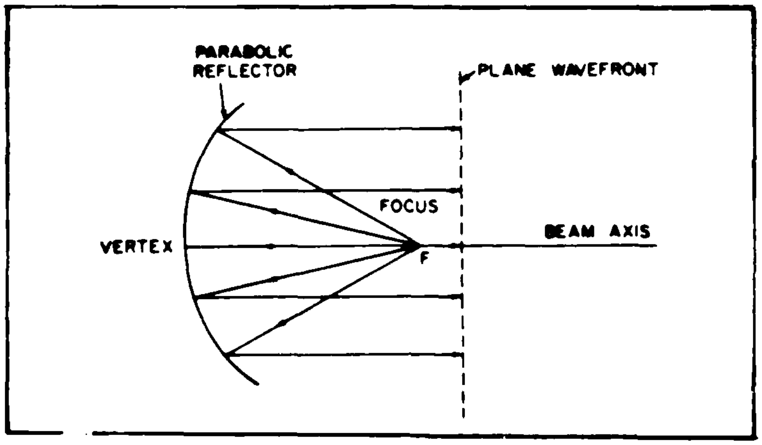

Figure 1. Operating Principle of a Paraboloidal Antenna (Skolnik, 1970:10-3)

operation. These excellent energy focusing and collimating properties are part of the reason why the paraboloid enjoys such widespread use.

Another reason for their popularity is relative design simplicity which, as stated above, is based upon geometric optics or ray tracing. Geometric optics is a high frequency approximation to Huygen's principle of physical optics; Huygen's principle states that "each point on a primary wavefront can be considered to be a new source of a secondary spherical wave and that a secondary wavefront can be constructed as the envelope of these secondary spherical waves". In contrast. geometric optics considers each point on the secondary wavefront to have a direct correspondence with one other point on the primary wavefront (Kraus, 1984:525,529). These ideas are illustrated in Figure 2.

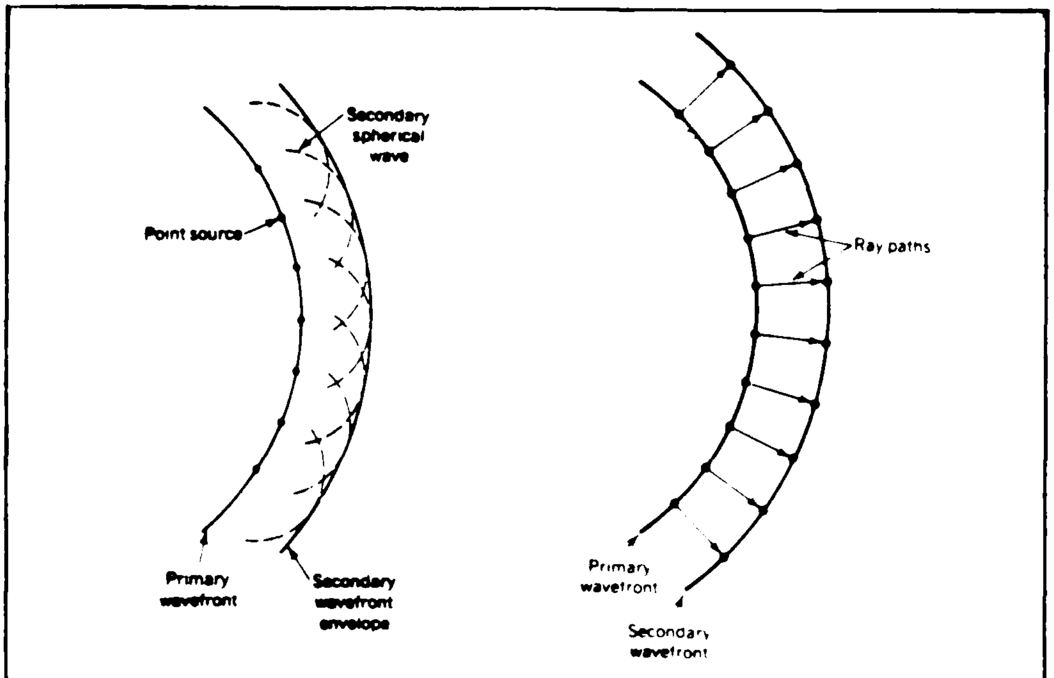

Figure 2. Secondary Wavefront Formation for Physical Optics and Geometric Optics (Kraus, 1984:525,530)

When applied to the analysis of paraboloids, which are typically many wavelengths across, geometrical optics can be used in conjunction with the aperture integration method to find the radiated field. Ray tracing is used to determine the distribution of the tangential field component over a plane that is normal to the axis of symmetry and that usually passes through the focal point. This surface is referred to as the aperture plane, and the field is assumed to be zero outside the projected area of the reflector onto this surface (Balanis, 1982:612). Integration over this surface and aperture distribution is then performed to find the radiated field. All ray paths from the focal point to the aperture plane are of equal length so, under ideal

conditions, the aperture illumination will be uniform in phase. This is not the case, however, with the amplitude portion of the distribution. The power density of spherical waves from the focal point falls off with the square of distance while plane waves present after reflection suffer no such spreading loss. Thus, components reflected near the center undergo less attenuation than those reflected nearer the edge, and a natural amplitude taper appears in the aperture distribution (Jull, 1981:66; Clarke and Brown, 1980:191). Careful design of the feed and shaping of the reflector surface can be used to increase or decrease this natural taper depending on the desired radiation pattern.

Fresnel Region Field Characteristics

The Fresnel region extends from a few wavelengths in front of the aperture plane out to the beginning of the far field at a distance of $2D^2/\lambda$. In the far field any point is essentially the same distance from all the points on the aperture plane; the radiation pattern is independent of range and power density varies simply with the inverse square of distance (Eq (1)). The Fresnel region "is characterized by the onset of diffusion outside the boundaries defined by the extension of the rays through the aperture" (Silver, 1949:172). All ray paths to a field point are no longer parallel so that constructive and destructive interference effects become important. The radiation pattern becomes dependent on range as well as

angular displacement with typical results being broadening of the main beam, blurring of sharp nulls, and an increase in sidelobe levels. These effects of finite range become more pronounced as range decreases. They can be accounted for mathematically by the addition of a quadratic phase term to the linear one present in the far field equation (Jull, 1981:31,34). Axial power density no longer varies with the inverse square of distance in this region, but fluctuates sinusoidally at an increasing rate as the range is diminished. Gain is, in general, degraded with aperture distributions that display the highest far field gain undergoing the most significant reduction (Hansen, 1976b:51; Jull, 1981:51).

Fresnel Zones

The division of a paraboloid's circular aperture plane into Fresnel zones (Figure 3) aids in understanding why the axial power density fluctuates sinusoidally in this region. When the observation point is in the far field it is nearly

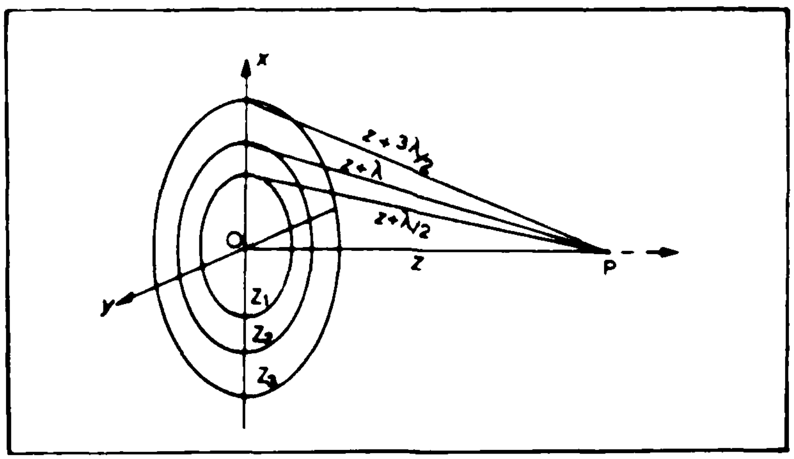

Figure 3. Fresnel Zones (Clarke and Brown, 1980:154)

the same distance from all points on the aperture plane, and there is no interference among aperture field components that are out of phase. As range decreases a point is reached where the path length difference between the aperture's center and edge is exactly $\lambda/2$. At this point the first Fresnel zone encompasses the entire aperture, and axial power density reaches its first maximum. As range decreases further, more and more zones are formed. The path length difference between a point on the edge of any zone and the aperture center is an integral number of half-wavelengths. Thus, for every contribution from one surface element in a particular zone, there is another one from an identical surface element in the next zone that is 180° out of phase. The result, as illustrated in Figure 4 for the case of uniform illumination, is alternating maxima and minima corresponding to, respectively, the formation of an odd or even number of Fresnel zones (Silver, 1949:196-197). (Note: This report will adopt the usual convention of presenting all plots of axial power density vs range with values of both normalized with respect to those at the far field distance of $2D^2/\lambda$ so that $\Delta = R/(2D^2/\lambda)$ in Figure 4.) There will be complete cancellation at certain axial ranges only for the particular case where the aperture distribution is uniform. If the antenna's aperture distribution

contains an amplitude taper, surface elements 180° out of phase will not have equal amplitudes, and the power density will reach a minimum instead of zero upon the formation of an even number of Fresnel zones.

Fresnel Approximations

Overview. Certain approximations are traditionally made to the integral equation describing the field resulting from a paraboloid's aperture distribution. The applicable geometry is illustrated in Figure 5. Kirchoff's scalar diffraction integral is the usual starting point and is given in a number of sources (Skolnik, 1970:9-7; Silver,

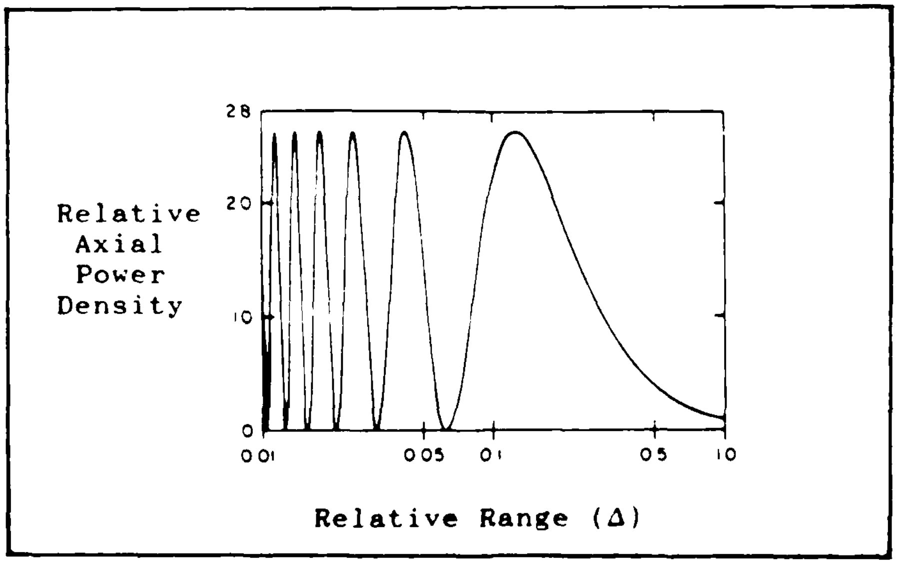

Figure 4. Power Density vs Range for Uniform Illumination (Jull, 1981:47)

1949:170). It asumes high frequency and linear polarization. If one also assumes uniform phase across the

aperture distribution, then this equation is given by

$$E_P = \int_0^{2\pi}\int_0^{a} G(\rho)\left[\frac{\exp(-jkr)}{4\pi r}\right][(jk+1/r)\cos(n,r)+jk]\ \rho\, d\rho\, d\phi' \qquad (3)$$

where

E_P = scalar field at observation point
$G(\rho)$ = aperture distribution
k = propagation constant, $2\pi/\lambda$
r = distance from aperture point to field point
(n,r) = angle between normal to aperture plane and the r direction

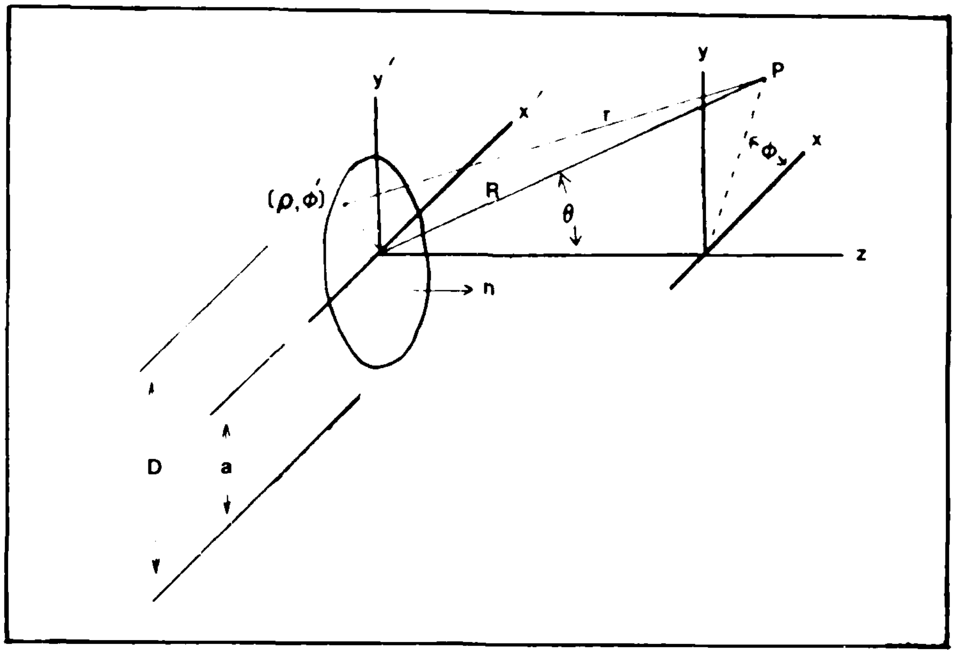

Figure 5. Aperture Geometry

The traditional approximations for the Fresnel region are the following (Skolnik, 1970:9-8):

1. The 1/r term inside the second set of brackets is negligible in comparison to k.

2. The 1/r term in the first set of brackets is set equal to the constant 1/R where R is the distance from the origin to the observation point.

3. The angle (n,r) is replaced by (n,R) = θ .

4. The r in the phase term exp(-jkr) is expanded in a power series and all terms higher than the second order are truncated so that (Hansen, 1964:26)

$$r = R - \rho\sin(\theta)\cos(\phi-\phi') + [\rho^2/2R][1 - \sin^2(\theta)\cos^2(\phi-\phi')] \quad (4)$$

5. If field points at wide angles are not considered then the Fresnel Small Angle (FSA) approximation neglects the $\sin^2(\theta)$ term in Eq (4) so that r in the phase term is usually approximated by (Hu, 1960:345)

$$r = R - \rho\sin(\theta)\cos(\phi - \phi') + \rho^2/2R \quad (5)$$

With these approximations the Kirchoff integral can be written as

$$E_P = B(\theta)\int_0^{2\pi}\int_0^a G(\rho)\exp\left[\frac{-jk\rho^2}{2R}\right]\exp[jk\rho\sin(\theta)\cos(\phi-\phi')]\rho d\rho d\phi' \quad (6)$$

where

$$B(\theta) = [1+\cos(\theta)]\ [jk/4\pi R]\ \exp(-jkR)$$

Eq (6) is the basis for the method developed in this report. This equation differs from the standard far field equation due to: (1) the addition of a quadratic phase term so that the $\pi/8$ phase criterion can be maintained and (2) the fact that the $[1+\cos(\theta)]$ term is not set equal to two.

Range of Accuracy. The Fresnel approximations will give accurate results for observation points which are not too close to the aperture plane and which are not too far off the antenna boresight. Close to the antenna the

approximations might fail due to consideration of phase errors or proximity to the reactive field at extremely close ranges (Hansen, 1964:30). However, only for extremely small antennas with diameters less than about eight wavelengths across does the reactive field consideration dominate (Skolnik, 1970:9-11). For axial displacements the approximations have been shown to give extremely accurate results as close as two diameters away from a uniformly illuminated paraboloid with a 100 wavelength diameter (Rahmat-Samii et al., 1981:162). More generally, the axial range restriction for a maximum phase error of $\pi/8$ has been given as

$$R > 0.5D(D/\lambda)^{1/3} \qquad (7)$$

which results in a minimum range of 1.1 diameters for a ten wavelength aperture and 2.3 diameters for a 100 wavelength aperture (Hansen, 1964:30; Skolnik, 1970:9-10). Eq (7) is the criterion for minimum range which is used in the method developed in this report.

At wide angles the cross-polarized field components cannot be neglected, and the FSA approximation mentioned earlier (Eq (5)) is no longer valid. However, since most of the energy is concentrated in a small angular region by practical paraboloids, the applicable angular range is usually sufficient. Even for antennas as small as ten wavelengths across, the FSA approximation gives accurate results for angles "covering up to four sidelobes in the far

field. Since this approximation is absolute with regard to angular range, the approximation improves greatly for large D/λ with regard to the regions of greatest interest near the principal lobe" (Rahmat-Samii et al., 1981:162). The general relationship between antenna dimensions, axial range, and maximum allowable angle off-axis is given by (Zucker, 1966:685)

$$\frac{\pi a^2}{\lambda z} \sin^2(\theta) \ll 1 \tag{8}$$

where

a = antenna radius
λ = wavelength
z = axial range
θ = angle off-axis

Eq (8) is always satisfied in the far field, but it requires that, in the Fresnel region, the observation point be restricted to smaller angles as the range decreases. If the relationship is made more specific so that

$$\frac{\pi a^2}{\lambda z} \sin^2(\theta) < 0.08 \tag{9}$$

then a maximum off-axis angle of 26.8^0 is allowed at the beginning of the far field where $z = 2D^2/\lambda$. In accordance with the findings of Rahmat-Samii et al. cited above, this allows coverage of almost four sidelobes of a ten wavelength antenna at that distance. Eq (9) is the criterion for maximum angular displacement used in the method developed in this report.

III. Literature Review

Overview

This chapter presents the results of a literature search on the subject of determining Fresnel region power density. The usual procedure followed by most investigators is to solve the scalar Kirchoff diffraction integral after the Fresnel approximations have been made (Eq (6)). Constant phase aperture illuminations that are rotationally symmetric in amplitude are usually assumed so that analysis is simplified and numerical integration can be performed with respect to only one variable. The method of presentation here is to show results that have been obtained assuming progressively more complex but realistic aperture distributions.

Uniform Illumination

Aperture distributions are usually chosen for the ease with which the resulting integration can be performed as well as for their similarity to practical distributions. A uniform amplitude distribution is not very realistic, but has been investigated extensively because it results in a field expression which can be evaluated in closed form. This illumination utilizes the paraboloid's physical aperture with the highest efficiency, and other distributions are usually evaluated relative to this case. In the far field it results in the highest gain and

narrowest beamwidth of any constant phase distribution but also results in the highest sidelobes, 17.57 dB.

The Fresnel region gain reduction on axis for a uniform amplitude distribution has been solved for analytically (Jull, 1981:47; Silver, 1949:199) and is given by

$$G/G_0 = [\sin(t)/t]^2 \tag{10}$$

where

G = Fresnel region gain
G_0 = far field gain at infinity
$t = ka^2/4z$
k = propagation constant
a = aperture radius
z = axial range

The graphical results of this expression have already been presented in Figure 4. Using Eq (10), the power gain at the beginnning of the far field ($z = 2D^2/\lambda$) is $0.99G_0$ where G_0 is the power gain at infinity.

Amplitude Taper Without Pedestal

This is the type of distribution (Eq (2)) utilized in the method currently used by the Air Force for calculating Fresnel region power density. The amplitude taper makes it more representative of practical distributions than the uniform illumination just discussed. However, as mentioned in Chapter 1, zero amplitude at the aperture edge does not usually happen. Nonetheless, this particular distribution has been widely used as a model for paraboloidal illumination.

A computer program has been written (Farrar and Adams, 1980:134-137) which uses this illumination to calculate Fresnel region power density. In contrast to the present Air Force method, this program does not restrict values of the taper control exponent to integer values, and they are estimated from the sidelobe ratio. Figure 6 shows the fluctuations in Fresnel region power density assuming this illumination and a 25 dB sidelobe ratio (Hansen, 1964:38).

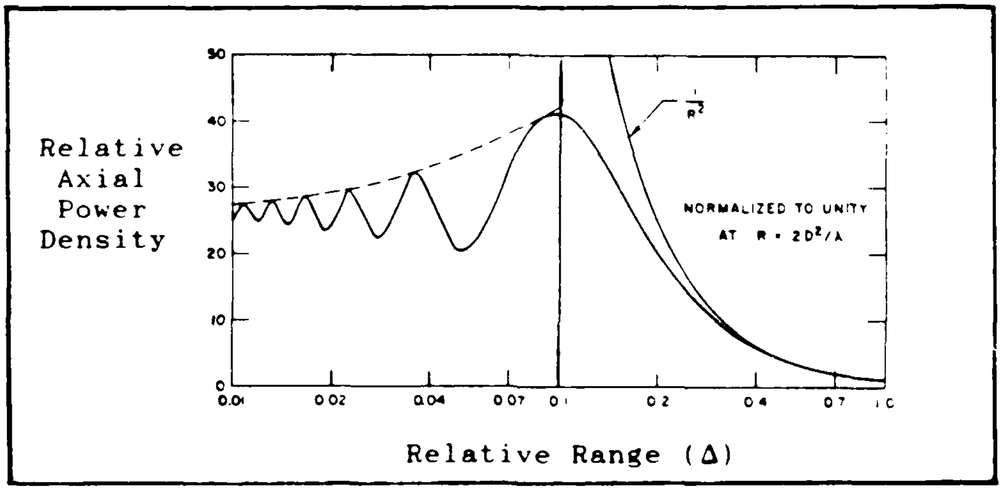

Figure 6. Power Density vs Range for Tapered Illumination Without Pedestal (Hansen, 1964:38)

Note the deviation from the simple monotonic increase as the Fresnel region is entered at $\Delta = 1$. Comparison of this figure with Figure 4 for the uniform case shows that, as expected, tapered illumination results in power density minima instead of zeros. Also, the envelope peak is not constant in the case of tapered distributions without pedestals.

Amplitude Taper With Pedestal

This type of aperture illumination is illustrated in Figure 7 and is an improvement over each of the two

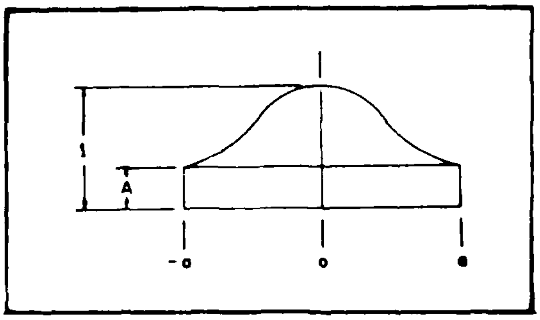

Figure 7. Aperture Illumination with Symmetric Taper on a Uniform Pedestal (Sciambi, 1965:79)

previously discussed distributions. The taper accounts for increased attenuation of components reflected near the paraboloid's edge, and the pedestal accounts for imperfect illumination of the reflector surface by the feed horn. One author has said:

> Of the various aperture distributions that have been studied, one in particular seems to satisfy most needs and covers typical behavior of actual dish antennas. This is the $(1-\rho^2)^N$ on a pedestal. The $(1-\rho^2)^N$ function has the important advantage that both pattern and directivity can be obtained in closed form. Inclusion of the pedestal adds versatility and allows closer representation of actual distributions. (Hansen, 1964:64)

The generic form of this illumination can be given as

$$G(x) = A + (1-A)(1-x^2)^N \tag{11}$$

where

$G(x)$ = normalized aperture distribution
A = amplitude of field at aperture edge
x = normalized aperture radial variable
N = exponent controlling rate of amplitude taper

The distribution has two independent parameters, A and N, and is normalized to one at the aperture center where $x = 0$ and tapers down to the non-zero value of A at the aperture edge where $x = 1$. The far field pattern resulting from an illumination of this form can be obtained by adding the patterns produced by the two component parts, i.e. the taper and the pedestal (Sciambi, 1965:79).

Hansen has developed a distribution which is different in form than Eq (11) but which also results in a symmetric amplitude taper on a uniform pedestal (Hansen, 1975:184; Hansen, 1976b:50-52; Hansen, 1976a:477-480). He was dissatisfied with the illumination in the form of Eq (11) because it "has two parameters, and there is no simple way of finding the values of these two parameters that optimize one quality, e.g. efficiency, for a given second quality, e.g. sidelobe ratio" (Hansen, 1975:184). He was interested in designing an illumination model that would permit circular aperture tradeoff studies based on a single parameter. The result of his work was the one-parameter circular aperture distribution of the form

$$G(x) = \frac{I_0[\pi H(1-x^2)]}{I_0[\pi H]} \qquad (12)$$

where

$G(x)$ = normalized aperture distribution
I_0 = modified Bessel function, zero order
x = normalized aperture radial variable
H = single parameter controlling illumination

This distribution is normalized to one at the aperture center and tapers off to the non-zero value of $1/I_0(\pi H)$ at the aperture edge. The H parameter is related to the sidelobe ratio by

$$slr = 17.57 \text{ dB} + 20\log\left[\frac{2I_1(\pi H)}{\pi H}\right] \tag{13}$$

where

slr = sidelobe ratio
I_1 = modified Bessel function, first order

It can be seen that the H parameter is uniquely related to the sidelobe ratio; once it is determined the aperture distribution is known. Beamwidth and aperture efficiency can also be determined through knowledge of the H parameter. Table I shows the theoretical one-parameter distribution values as determined by Hansen.

This one-parameter aperture distribution has been used by Hansen (Hansen, 1976b:50-52) to calculate Fresnel region power density. He concluded that the resulting curves "should more closely represent actual parabolic reflector and circular array antennas" than those resulting from assuming a symmetric taper without a pedestal and that "they are appropriate for use in calculations of near field radiation hazards" (Hansen, 1976b:52). These curves are reproduced in Figures 8 and 9. Note that in comparison to Figure 6, where the aperture distribution tapered to zero at the edge, inclusion of an amplitude pedestal results in

Table I

One-Parameter Distribution Values (Theoretical)

Sidelobe Ratio (dB)	H	Illumination Edge Taper (dB)	Half-Power Beamwidth (radians)	Aperture Illumination Efficiency
17.57	0	0	1.0290 λ/D	1.0000
20	0.4872	4.49	1.0787	0.9786
25	0.8899	12.35	1.1739	0.8711
30	1.1977	19.29	1.2607	0.7595
35	1.4708	25.78	1.3403	0.6683
40	1.7254	31.98	1.4139	0.5964
45	1.9681	38.00	1.4827	0.5390
50	2.2026	43.89	1.5474	0.4923

(Hansen, 1976a:479)

oscillations of nearly constant amplitude. Also, as mentioned in Chapter 2, the less tapered distributions, which result in the greatest far field gain and highest sidelobes, undergo the most severe Fresnel region gain reduction.

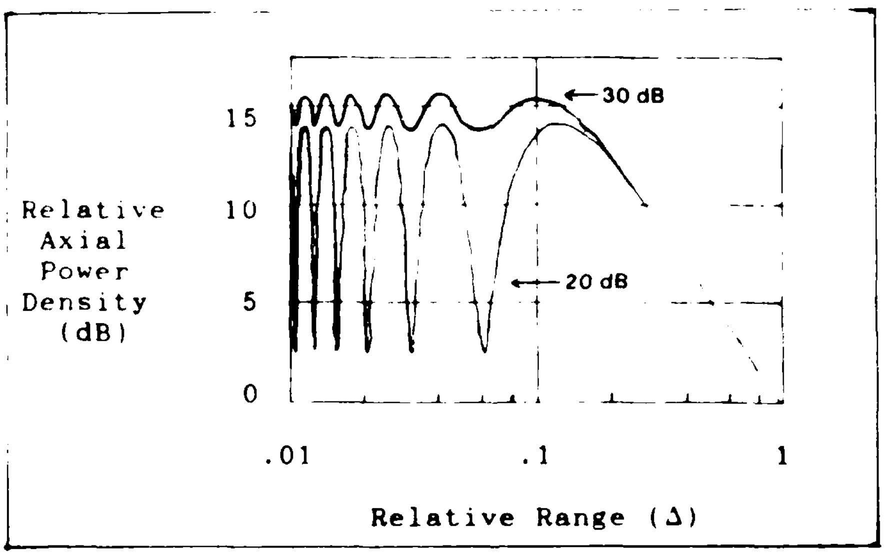

Figure 8. Power Density vs Range for the One-parameter Aperture Distribution and 20/30 dB Sidelobes (Hansen, 1976b:50)

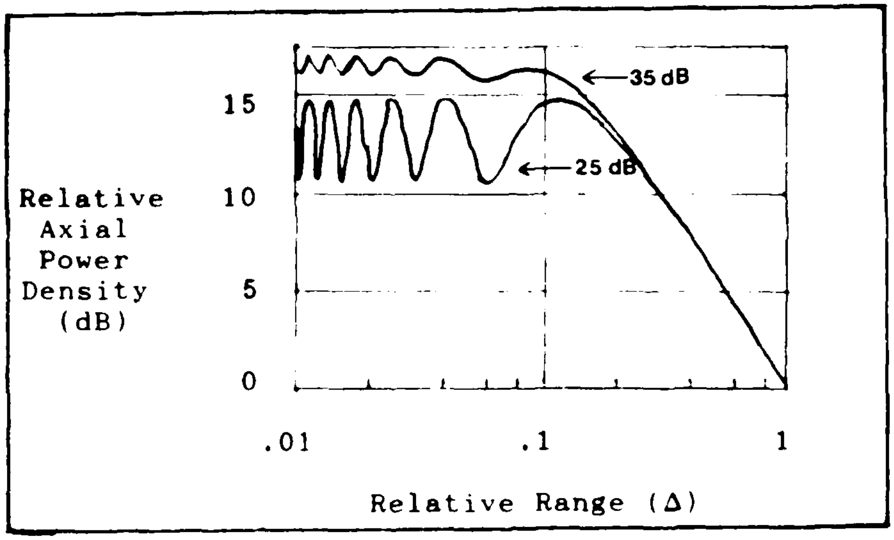

Figure 9. Power Density vs Range for the One-parameter Aperture Distribution and 25/35 dB Sidelobes (Hansen, 1976b:50)

IV. Design and Structure of POWERDEN

Overview

This chapter presents the details of how POWERDEN (Appendix A) works in both its axial and off-axis mode. Basic relationships for determining the power density are discussed along with the Romberg numerical integration technique which is used. Limitations on aperture size, axial range, and off-axis range are presented followed by a discussion of the program's input requirements and output information format. The chapter concludes with a discussion of some of the practical aspects of using POWERDEN.

Basic Relationships

Power Density. If an observation point is in the far field or Fresnel region, the reactive power is negligible, and the power density is proportional to the square of the field's magnitude at that point (Silver, 1949:113) or

$$S \sim \left| E_P \right|^2 \tag{14}$$

where

S = power density
E_P = field at observation point

POWERDEN calculates the power density according to Eq (14) at discreet points within the Fresnel region using the one-parameter circular aperture distribution. Since this illumination model is normalized to one, the actual power

density at these points is obtained by, in turn, normalizing the calculated values with respect to a known quantity, i.e. the power density on axis at a range of $2D^2/\lambda$. This distance marks the beginning of the far field so its value is readily determined by Eq (1).

<u>Field Strength</u>. The field's relative magnitude at the observation point is given by

$$\left|E_P\right| \sim \frac{1 + \cos(\theta)}{\Delta} \left| \int_0^1 G(x)\ J_0(ux)\ \exp\left[\frac{-j\pi x^2}{8\Delta}\right] x\ dx \right| \qquad (15)$$

where

$\Delta = R/(2D^2/\lambda)$, relative range
$G(x) = I_0[\pi H(1-x^2)]$, relative aperture illumination
I_0 = modified Bessel function, zero order
H = single parameter developed by Hansen
J_0 = Bessel function, zero order
$u = ka\sin(\theta)$
$k = 2\pi/\lambda$, propagation constant
a = aperture radius
$x = \rho/a$, normalized aperture radial variable

Eq (15) results from Eq (6) after performing the integration with respect to ϕ' and making the substitutions for the normalized aperture radius and range (Appendix D). The value of this expression is obtained by numerical integration and then squared to result in the pre-normalized value of power density as discussed in the last section.

<u>H Parameter</u>. The H parameter is related to the sidelobe ratio by Eq (13) which is reproduced here in a different form:

$$slr - 17.57dB - 20\log\left[\frac{2I_1(\pi H)}{\pi H}\right] = 0 \qquad (16)$$

It is solved for by a subroutine which utilizes the Newton-Raphson method (Chapra and Canale, 1985:137-143) for finding the root of Eq (16). An initial estimate based on Hansen's findings (Table I) is generated internally within the program, and the routine converges quadratically for sidelobe ratios up to 95 dB.

Numerical Integration

The real and imaginary parts of Eq (15) are integrated numerically using the Romberg adaptive trapezoidal technique (Chapra and Canale, 1985:416-424). An adaptive method was chosen so that the minimum number of divisions would be used at each observation point. The integral converges more rapidly for points near the axis and far from the antenna; use of a method such as Simpson's, where a fixed number of divisions is used, would result in wasted time during evaluation at these points. The number of divisions is continually doubled up to a maximum of 4096 (2^{12}) or until an error limit set by the operator is reached. All plots in this report that are based on data generated by POWERDEN were arrived at using the default error limit of 0.1%

Limitations

Aperture Diameter. The method developed in this report is valid only for electrically large antennas. The scalar Kirchoff diffraction integral (Eq (3)) is based on this assumption as is the design of paraboloids by the laws of geometric optics. POWERDEN uses a value of ten

wavelengths for the minimum aperture diameter. Most practical paraboloids are much larger than this at typical microwave frequencies. Below this limit the high frequency assumptions may not be valid, and, at close range, amplitude considerations due to proximity to the reactive field will dominate phase considerations when trying to establish a minimum axial range (Eq (7)).

Axial Range. The Fresnel approximations which Eq (15) is based on are not valid for all of the Fresnel region. Just as the far field of a large paraboloid begins farther away from the aperture plane than that of a smaller one, the minimum range at which the Fresnel approximations are accurate is greater for a large antenna. As mentioned in Chapter 2, the criterion for minimum range is

$$R > 0.5D(D/\lambda)^{1/3} \tag{7}$$

In the axial mode of operation POWERDEN calculates power density at discreet points starting at the beginning of the far field ($\Delta = 1$) and proceeding toward the paraboloid along the boresight until the minimum distance (Eq (7)) is reached or until a relative range of $\Delta = 0.01$ is reached, whichever is greater. The latter condition will prevail for antennas wider than 125λ. As is evident from an examination of Figures 8 and 9, at distances less than $\Delta = 0.01$, power density is varying so rapidly with respect to distance that, in terms of establishing whether the power density is at a hazardous level, there is little need to go farther.

Off-axis Range. The FSA approximation discussed in Chapter 2 is valid for angles up to about 30^0 off the axis of symmetry (Hansen, 1964:27,30,40). This range covers many sidelobes for a large antenna near the far field and covers the first four sidelobes of a ten wavelength antenna with uniform illumination at a distance of $2D^2/\lambda$. The criterion used by POWERDEN for limiting off-axis range was given in Chapter 2 as

$$\frac{\pi a^2}{\lambda z} \sin^2(\theta) < 0.08 \qquad (9)$$

A value of 0.08 in this equation allows coverage of up to 26.8^0 at the far field limit of $2D^2/\lambda$. It has been shown (Rahmat-Samii et al., 1981:162) that the FSA approximations give extremely accurate results within this angular range even for a paraboloid as small as ten wavelengths in diameter.

Program Input

Axial Mode. The axial mode is used to determine if a hazardous power density level exists along the antenna boresight and, if so, what the maximum value is and at what range it occurs. In this mode of operation POWERDEN requires the following user input:

1. Sidelobe Ratio: This is the ratio of the main beam power density to that of the first sidelobe. It is

given in decibels and is limited to a minimum of 17.57 dB for a uniform illumination and a maximum of 95 dB. This maximum value is much larger than values associated with typical paraboloidal antennas. The sidelobe ratio is used by POWERDEN to calculate the H parameter so that the aperture distribution is fixed (Eq (12)).

2. Operating Frequency: Input in GHz.

3. Antenna Diameter: This is limited to a minimum value of ten wavelengths and is input in units of meters.

4. Gain and Transmitted Power: These values are used to calculate the power density (Eq (1)) at a range of $2D^2/\lambda$. This figure is needed for determination of absolute power densities since values calculated internal to POWERDEN are normalized with respect to those at this distance. Gain is input in decibels and power in kilowatts.

5. Maximum Numeric Integration Error: The Romberg integration technique utilized by POWERDEN compares the most recent value of the integral with that computed using half as many divisions during the previous integration. The number of divisions continues to double up to a maximum of 4096 (2^{12}) or until the two most recently calculated values differ by no more than this error limit. The default error limit is set at 0.1%.

6. Output to Line Printer or Disc File: Data output to a disc file might be desirable either if the line printer is being used or if it cannot be allowed to stand idle waiting for POWERDEN to complete its calculations. If

disc file output is selected, POWERDEN prompts the operator for a filename. This filename must be a valid DOS path.

Off-axis Mode. The off-axis mode is used for calculating power density at points along a line perpendicular to the axis of symmetry starting at some axial range greater than the limit imposed by Eq (7). The goal is the determination of a hazard radius outside of which the power density is below an acceptable level. In this mode of operation POWERDEN requires all of the same inputs just given for the axial mode plus the following:

1. Off-axis Starting Point: This will probably be determined from information obtained using POWERDEN in the axial mode as well as the potentially hazardous situation which is being analysed. The input is given in units of meters.

2. Off-axis Stopping Point: The optimum value of this input depends strictly on the individual antenna being analysed and the experience of the operator. The stopping point will be a value which can be assumed by the expression

$$u = ka\sin(\theta) \tag{17}$$

where

$k = 2\pi/\lambda$, propagation constant
a = antenna radius
θ = off-axis angle

Construction of off-axis radiation patterns in terms of u suppresses the effect of diameter size so that the patterns are dependant only on aperture illumination and range. The

default value is 16, and this will cover about four sidelobes at the beginning of the far field of a uniformly illuminated aperture. If power density at the off-axis starting point is only slightly above the applicable standard, it probably would not be necessary to go out to an angle where u = 16. On the other hand, an extremely high power density at the starting point might require a larger angular coverage so that u should be greater than 16.

Program Output

Axial Mode. Using POWERDEN in the axial mode results in the following data (Appendix B) being output to a line printer or disc file:

1. Relative Range: POWERDEN starts at a relative range of $\Delta = 1$ and calculates the real and imaginary parts of Eq (15) for 196 points along the axis until $\Delta = 0.01$. This assumes the range criterion given by Eq (7) is not violated. These points are not evenly distributed between the two end points; smaller steps are used after $\Delta = 0.1$ and even smaller ones after $\Delta = 0.02$. This greater resolution insures that the increasingly rapid variation in power density at closer ranges (Figures 8 and 9) is in fact reflected by the output data.

2. Relative Power Density: This is the ratio expressed in decibels of power density at the particular point on axis to power density at the beginning of the far field where $\Delta = 1$. These relative power density figures

make the detection of maximums and minimums easier and, in conjunction with the corresponding figures for relative range, make possible the production of plots such as Figures 8 and 9. If either the real or imaginary part of the integral in Eq (15) fails to converge at a point, then no relative power density information is given for that point.

3. Absolute Range: This is the actual distance in meters between the origin of the aperture plane and the point at which power density has been calculated.

4. Absolute Power Density: This is the actual power density in milliwatts per square centimeter at the particular axial point. As with the relative power density, no information is given if the integral did not converge at a particular point.

Off-axis Mode. Use of POWERDEN in the off-axis mode results (Appendix C) in the output of absolute power density information in the same manner as with the axial mode. The other output is the following:

1. Value of kasin(θ): POWERDEN proceeds off-axis in increments of u = 0.1 and continues at this same rate until the limit set by the operator is reached.

2. Relative Power Density: This information differs from that given in the axial mode in that the ratio is with respect to power density at the off-axis starting point rather than the beginning of the far field. This information along with the corresponding values of u makes possible the production of off-axis radiation plots of the

kind introduced in the next chapter. If either part of the integral fails to converge for the off-axis starting point, relative power density cannot be given for any of the subsequent points since all of them are normalized with respect to the value at this starting point. Absolute power density data, however, will still be available.

3. Distance Off-Axis: This is the distance in meters from the field point to the antenna boresight along a line perpendicular to the boresight. This information allows determination of the radiation hazard radius.

Practical Aspects

On-screen Information. While POWERDEN is running, information about the calculations at each data point is displayed. A typical set of information for an axial observation point might be

```
 2    1.688104E-02
 4   -3.990152E-03
 8    2.073011E-02
16    2.074877E-02
Re          0.8400
```

The first four lines show the number of divisions used by the Romberg integration routine and the corresponding integration results. Once the error limit set by the operator has been reached, the last line is displayed and shows that this calculation is for the real part of the integral at a relative range of Δ = 0.8400. The imaginary part of the integral is computed after the real part has

been determined for all points and, in that case, the letters "Im" are displayed instead of "Re". An on-screen information set is the same for the off-axis mode except the last line shows the value of u instead of Δ.

Running Time. Like any computer program, the time required by POWERDEN to complete a set of calculations depends to a large degree on the characteristics of the computer, i.e. clock speed, etc. The computer used to produce the data used for this report was a government owned Z-248 (AT compatible) equipped with a numerical math coprocessor (80287 chip). With this configuration, typical run times for both axial and off-axis modes were in the 10-20 minute range depending in the antenna parameters and numerical integration error limit. The math coprocessor is highly recommended since its absence can result in a tenfold increase in run time.

V. Results

Overview

This chapter presents in graphical form the results of applying POWERDEN to the analysis of a uniform aperture illumination (H = 0) and to four others in the form of the one-parameter distribution (Eq (12)) with assumed sidelobe ratios of 20, 25, 30, and 35 dB. Axial results are compared directly with those achieved by Hansen. Off-axis results for the one-parameter distribution have not been published so direct comparison is not possible except in the far field case of the uniform distribution since it is available in most books dealing with paraboloidal analysis. Therefore, off-axis results for the four other tapered distributions will be presented and analysed in relation to what is expected theoretically. The goal will be to establish that POWERDEN yields reasonable data and can be a useful tool for determining hazardous radiation zones in the Fresnel region.

Axial Data

Figures 10 and 11 show the results of using POWERDEN in the axial mode for the analysis of the four tapered distributions. Direct comparison with Hansen's published results (Figures 8 and 9) shows that POWERDEN achieves nearly identical results. The less tapered distributions with the highest sidelobes undergo the most significant near field gain reduction. Power density fluctuations begin

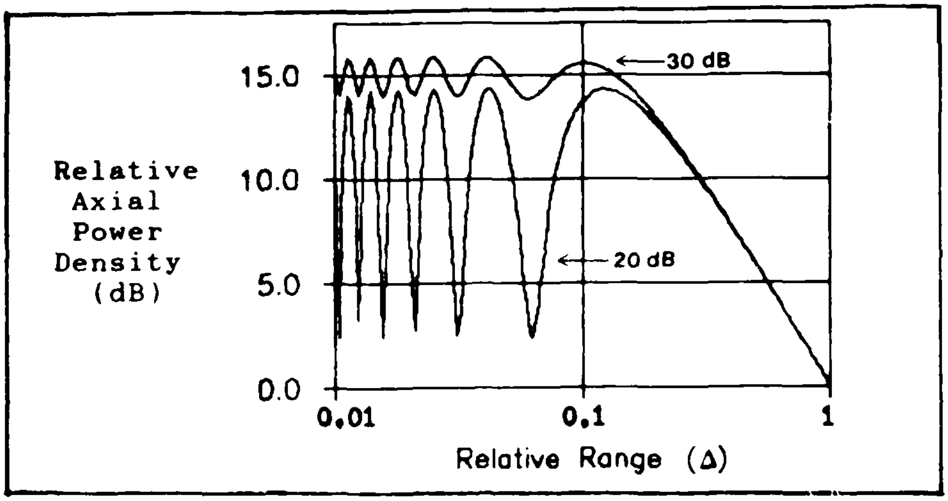

Figure 10. POWERDEN Axial Radiation Pattern for Sidelobe Ratios of 20/30 dB

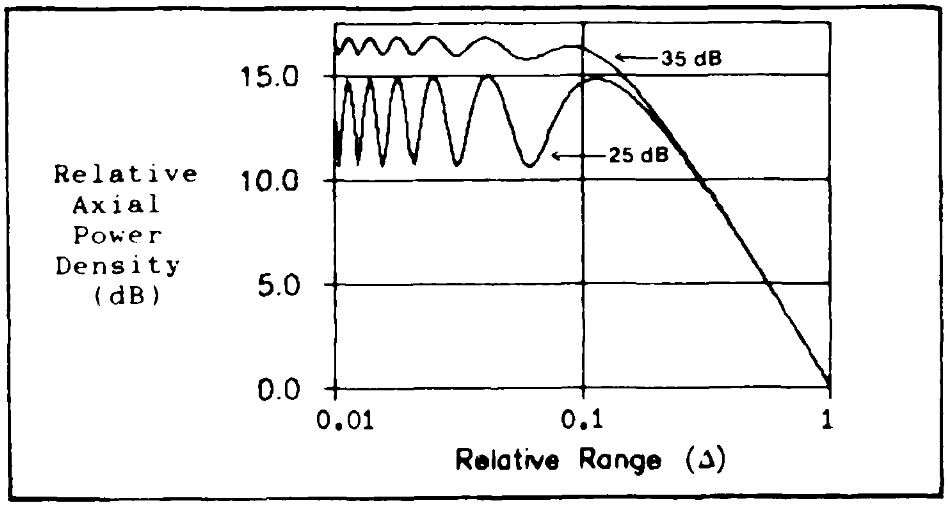

Figure 11. POWERDEN Axial Radiation Pattern for Sidelobe Ratios of 25/35 dB

around a relative range of $\Delta = 0.1$ as the aperture begins to encompass an increasing number of Fresnel zones. The magnitude of the fluctuations decreases as the illumination becomes more highly tapered; this is due to the decreasing field strength of the outer Fresnel zones in a tapered

illumination compared to the inner ones. In contrast to Figure 6 where the aperture distribution went to zero at the paraboloid's edge, i.e. no pedestal, here the oscillation peaks are nearly constant for a given sidelobe ratio. (Hansen, 1976b:51)

Off-Axis Data

Far Field. Radiation patterns resulting from use of POWERDEN in the off-axis mode at a single range of $2D^2/\lambda$ are presented in this section so that the effect of increasing illumination taper can be seen without being obscured by near field phenomena. Increasing taper should, in general, result in wider beamwidth, lower gain, and lower sidelobe levels (Silver, 1949:179; Jull, 1981:22).

Figure 12 is the theoretical far field radiation pattern of a uniform distribution while Figure 13 was produced from POWERDEN output data. It can be seen that the two figures

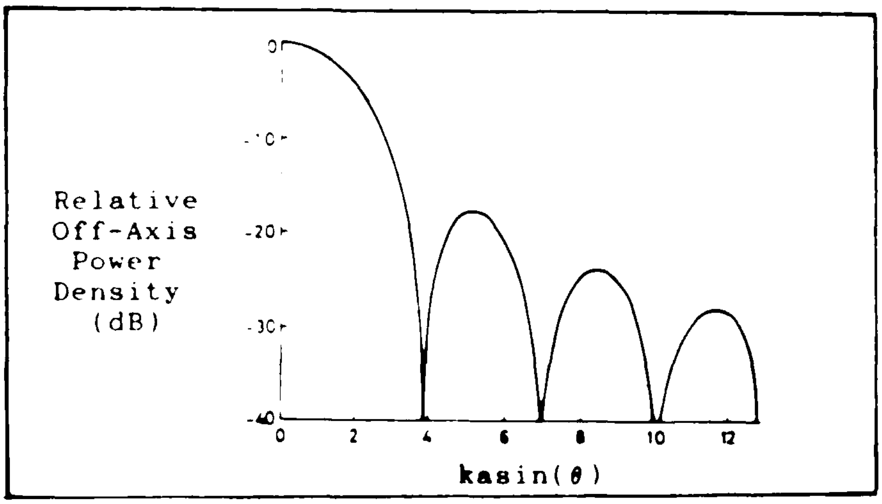

Figure 12. Theoretical Far Field Radiation Pattern of a Uniformly Illuminated Paraboloid (Rudge, et al., 1982:43)

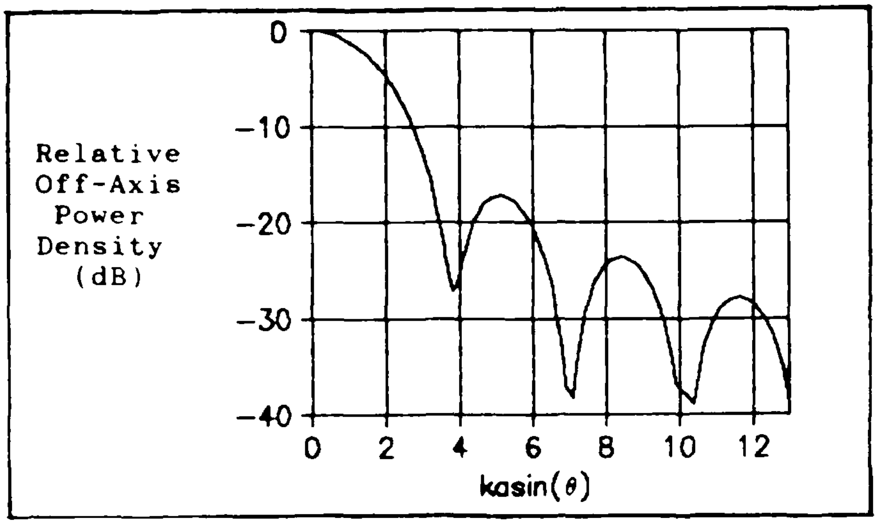

Figure 13. POWERDEN Far Field Radiation Pattern of a Uniformly Illuminated Paraboloid

correspond closely with respect to their major features. The theoretical pattern is produced from the closed form solution

$$F(u) \sim \left[\frac{J_1(u)}{u} \right] \tag{18}$$

where u is as defined in Eq (17) and J_1 is the Bessel function of the first order. The slanted lines in the location of the nulls of Figure 13 and other POWERDEN off-axis plots in this report are present because the data points fell below the -40 dB level.

Figures 14-17 result from POWERDEN applied to the far field of the four tapered distributions. Note that, as expected, the width of the main beam increases, and the level of the first sidelobe drops as the illumination becomes more highly tapered. However, the sidelobe levels

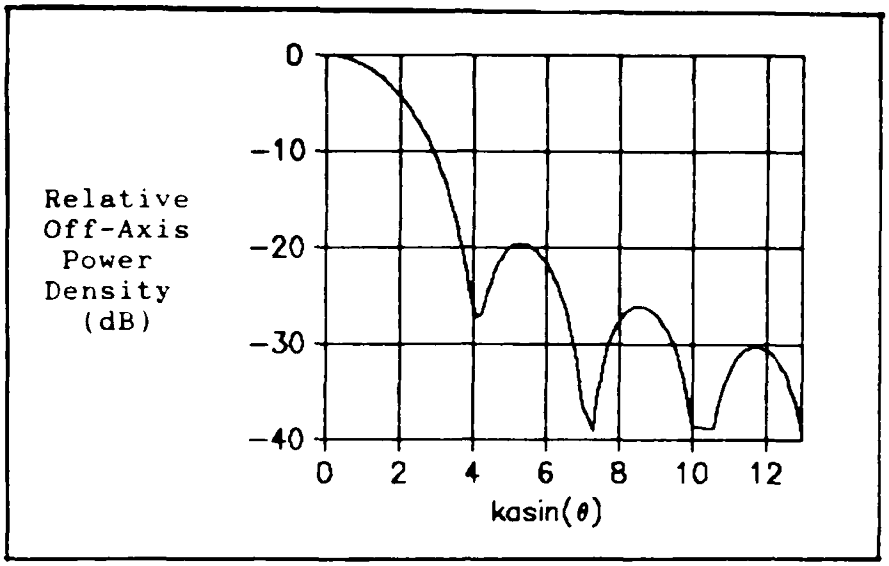

Figure 14. POWERDEN Far Field Radiation Pattern (slr = 20 dB)

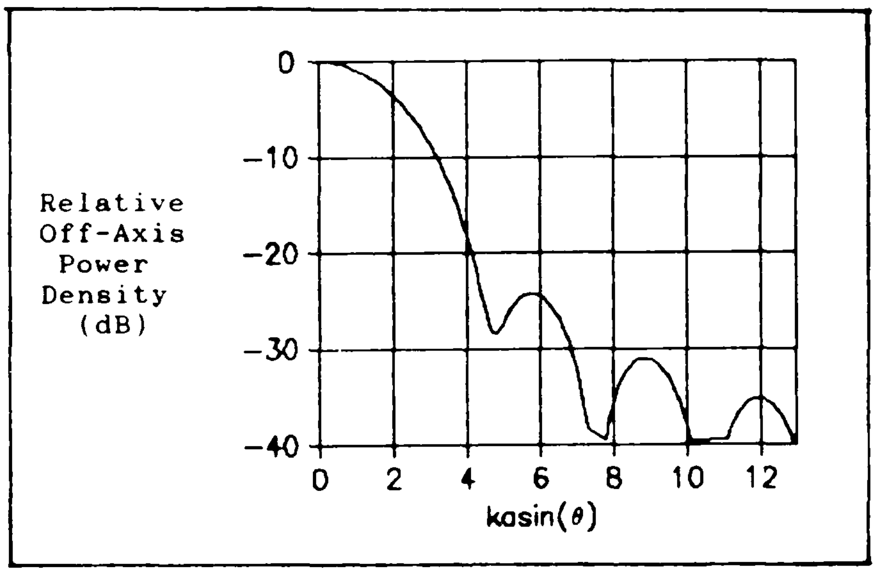

Figure 15. POWERDEN Far Field Radiation Pattern (slr = 25 dB)

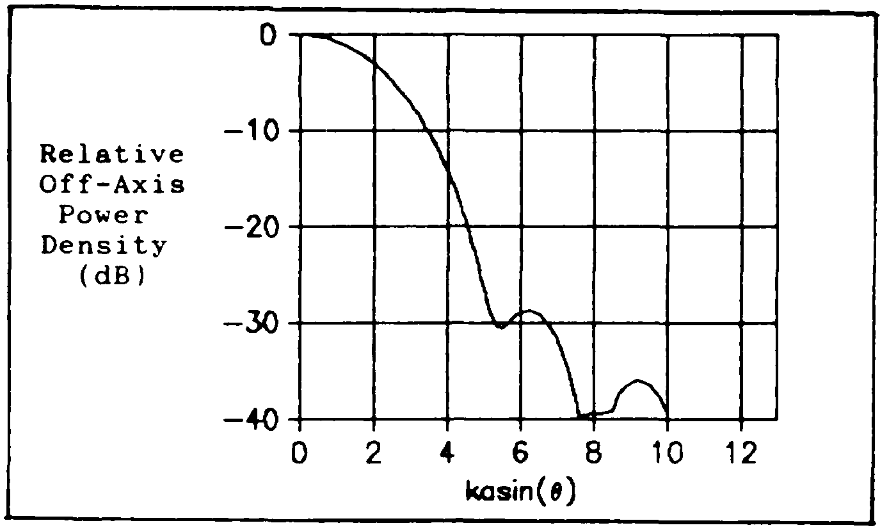

Figure 16. POWERDEN Far Field Radiation Pattern (slr = 30 dB)

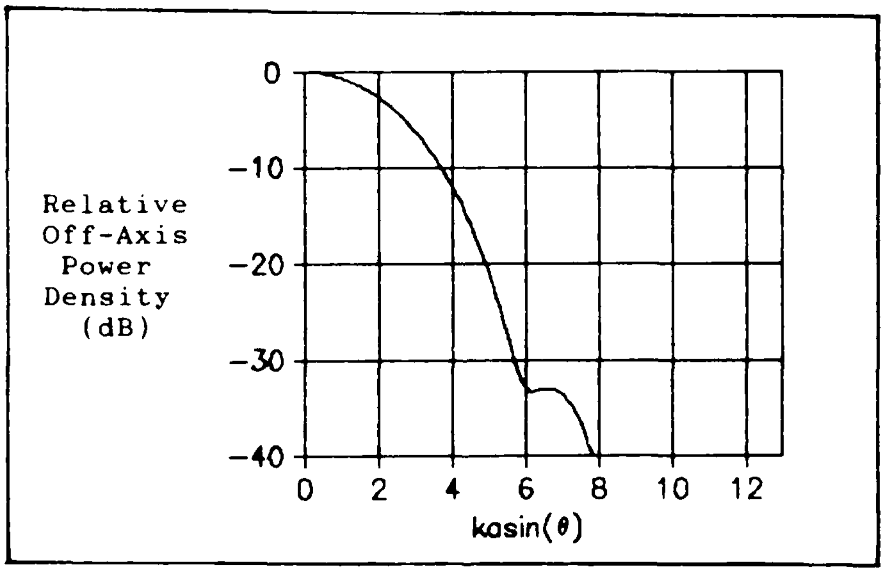

Figure 17. POWERDEN Far Field Radiation Pattern (slr = 35 dB)

do not correspond exactly to those which were initially input into the program so that the H parameter could be calculated. Each is slightly higher with the effect increasing along with illumination taper. For instance,

Figure 16 displays a sidelobe level of about 29 dB instead of the 30 dB which was input. Beamwidths are also slightly larger than those given in Table I. The deviations from theoretical expectations are quantified in Table II. The POWERDEN sidelobe ratios are those resulting from the

Table II

Comparison of Theoretical One-Parameter Distribution Values With Those Resulting From POWERDEN

Input Sidelobe Ratio (dB)	POWERDEN Sidelobe Ratio (dB)	Beamwidth From Table I (radians)	POWERDEN Beamwidth (radians)
17.57	17.31	1.0290 λ/D	1.0295 λ/D
20	19.63	1.0787	1.0800
25	24.30	1.1734	1.1758
30	28.78	1.2607	1.2635
35	33.00	1.3403	1.3437

resolution provided by incrementing the angular range in steps of $u = 0.1$, and the POWERDEN beamwidths were computed by linear interpolation between the two values of u which fell on either side of -3.01 dB. Although the sidelobe levels are elevated, the practical effect will be the establishment of a wider radiation hazard radius than is absolutely necessary. This same observation can be made regarding the slightly wider beamwidths; the theoretical half-power point is closer to the antenna boresight so the radiation hazard radius will be slightly larger than called for in the ideal case.

Fresnel Region. The quadratic phase term in Eq (15) was included so that the phase criterion of $\pi/8$ used in the far field could be maintained in the Fresnel region. The effect of finite range has been described as follows:

> In the region closer than $2D^2/\lambda$, called the radiating near field region, the pattern varies with the distance and, due to the quadratic phase, perfect interference does not occur. This blurs the nulls and raises the sidelobes. At closer distances ... the main beam broadens and some close-in sidelobes may be swallowed by the main beam. (Hansen, 1964:29)

In addition, the quadratic phase has a greater effect on the first null than on subsequent ones (Rusch and Potter, 1970:99), and it has a smaller overall effect on the more highly tapered distributions (Jull, 1981:32).

Figures 18 and 19 illustrate these effects for the case of a rectangular aperture with, respectively, uniform and tapered illumination. The scales are different than those used for other figures in this report, but they do illustrate the main effects of finite range ($\beta = 0$ is at a relative range of $\Delta = \infty$ while $\beta = \pi/2$ is at a relative range of $\Delta = 0.25$). The sidelobes are elevated for $\Delta = 0.25$, and each figure shows a broadened main beam which, in Figure 19 for the tapered distribution, has completely swallowed the first sidelobe. In addition, the first null in each far field plot is blurred more than the second.

Figures 20-24 show the off-axis results achieved by POWERDEN in the Fresnel region of the uniform and tapered distributions. For each illumination results are plotted for

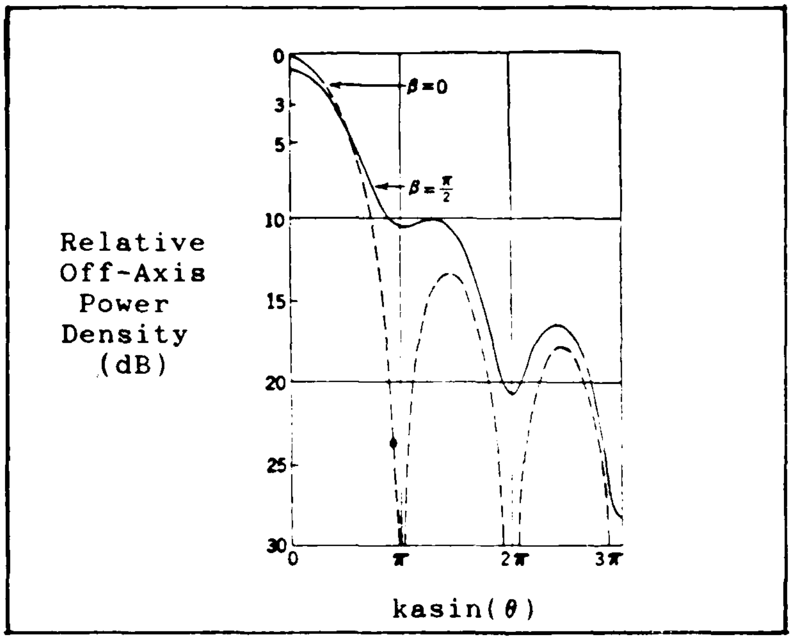

Figure 18. Near Field Effect for a Rectangular Aperture with Uniform Illumination (Silver, 1949:190)

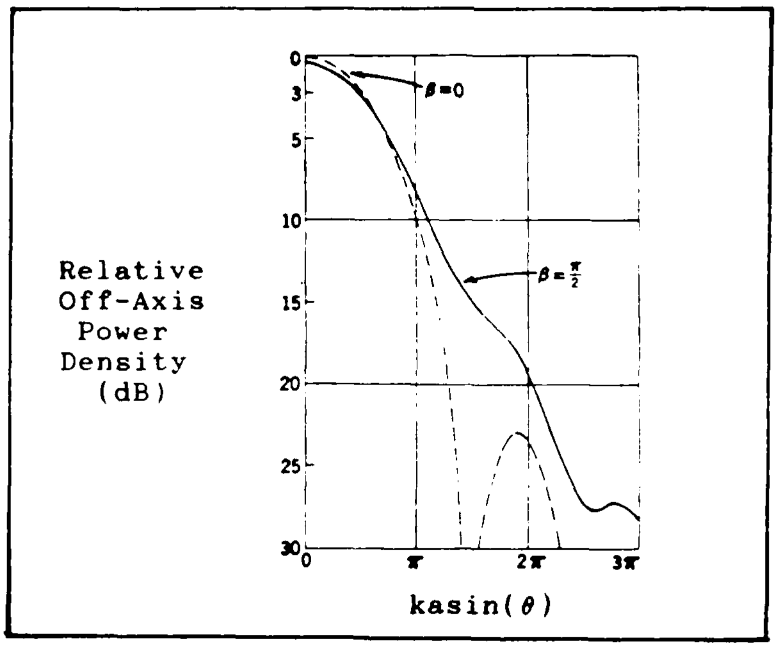

Figure 19. Near Field Effect for a Rectangular Aperture with Tapered Illumination (Silver, 1949:190)

relative ranges of Δ = 1, Δ = 0.5, and Δ = 0.25. Note that, as predicted, the sidelobes are elevated with decreasing range and the beamwidths are broadened. At Δ = 0.25 the 30 and 35 dB sidelobes are nearly totally engulfed by the main beam. In each of the five cases the first null is affected more significantly than is the second. The more highly tapered distributions with the larger sidelobe ratios suffer less overall distortion from the quadratic phase effects of the closer ranges.

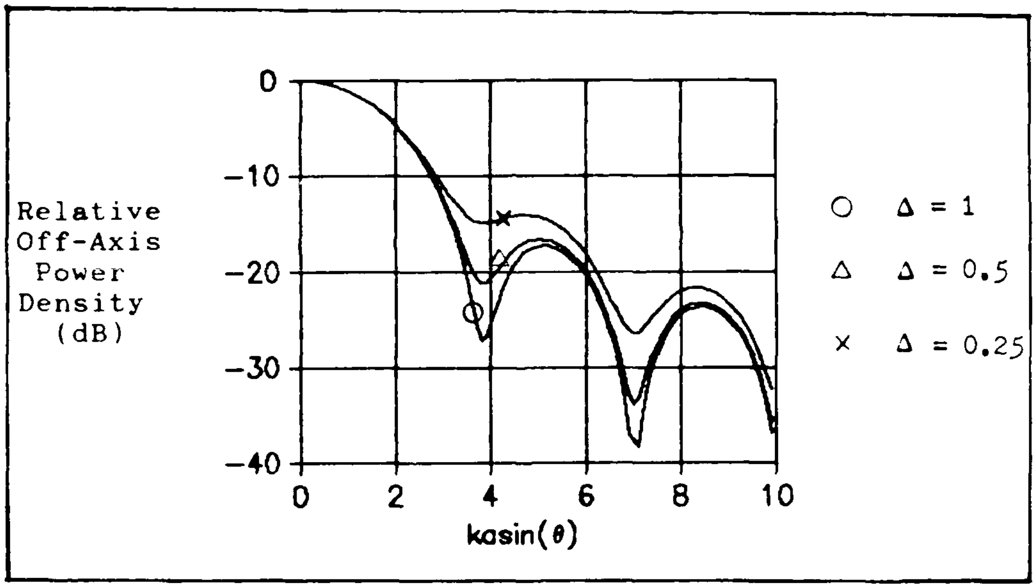

Figure 20. POWERDEN Fresnel Region Radiation Pattern (slr = 17.57 dB)

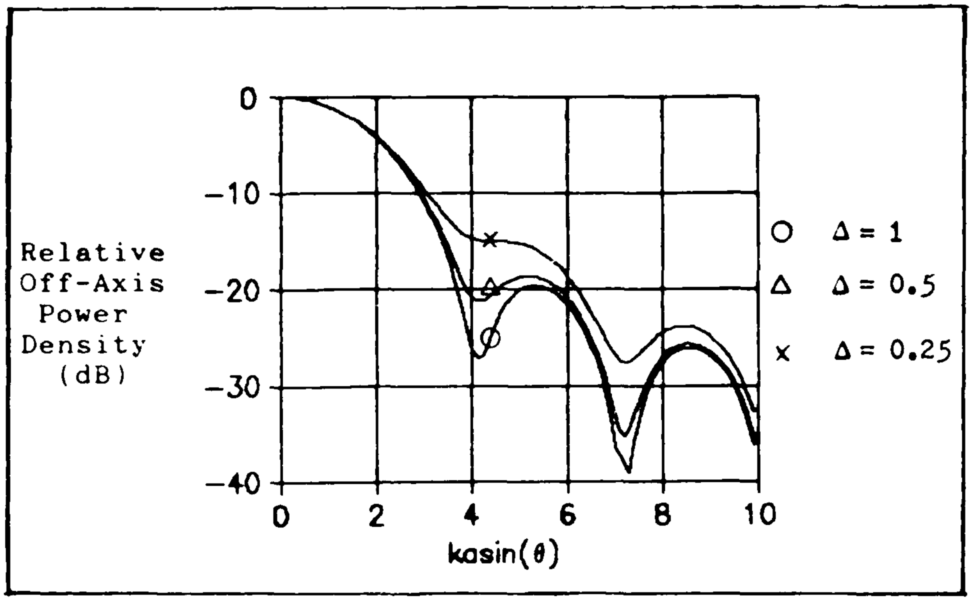

Figure 21. POWERDEN Fresnel Region Radiation Pattern (slr = 20 dB)

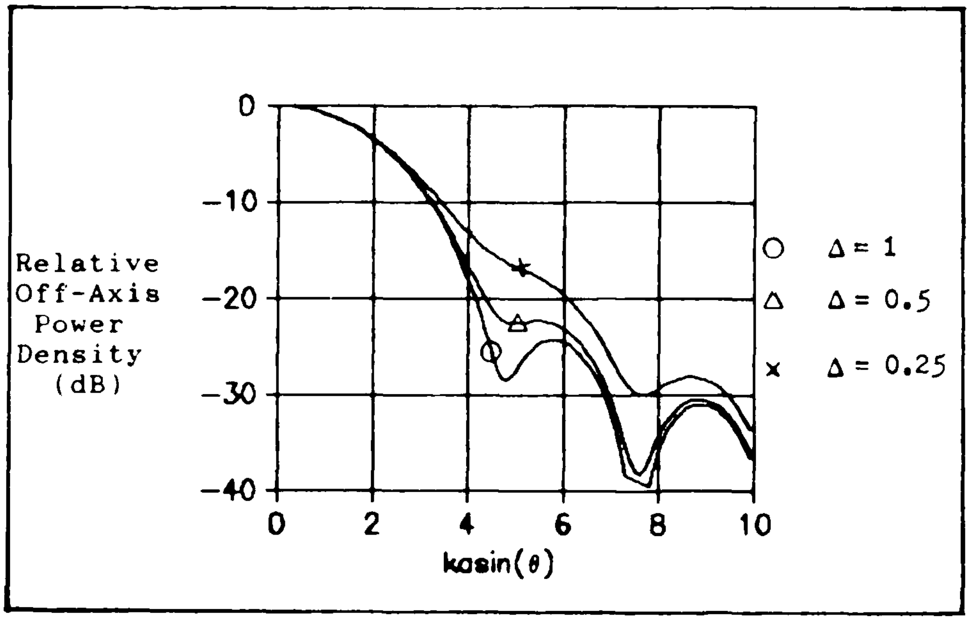

Figure 22. POWERDEN Fresnel Region Radiation Pattern (slr = 25 dB)

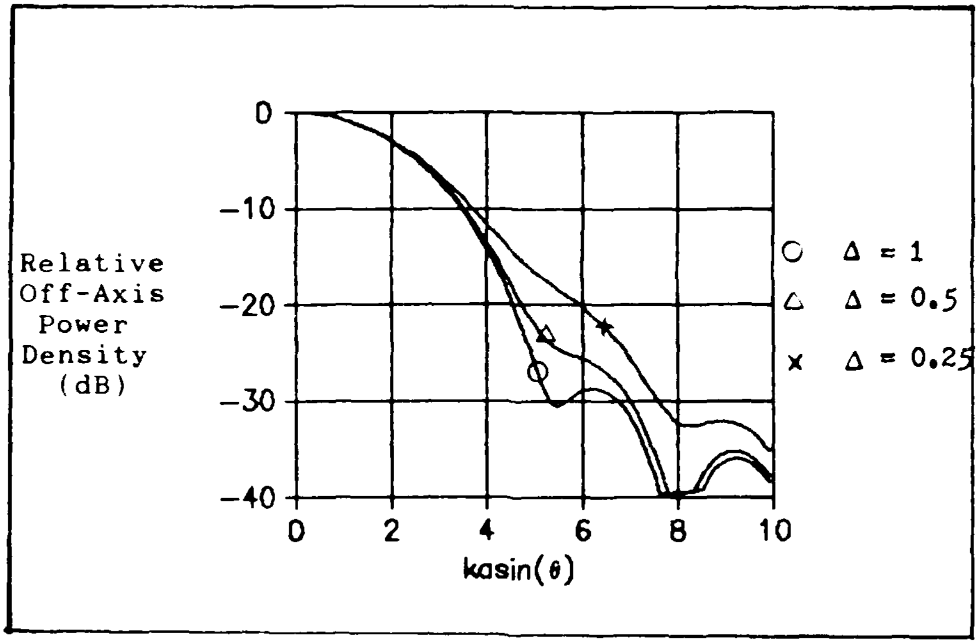

Figure 23. POWERDEN Fresnel Region Radiation Pattern (slr = 30 dB)

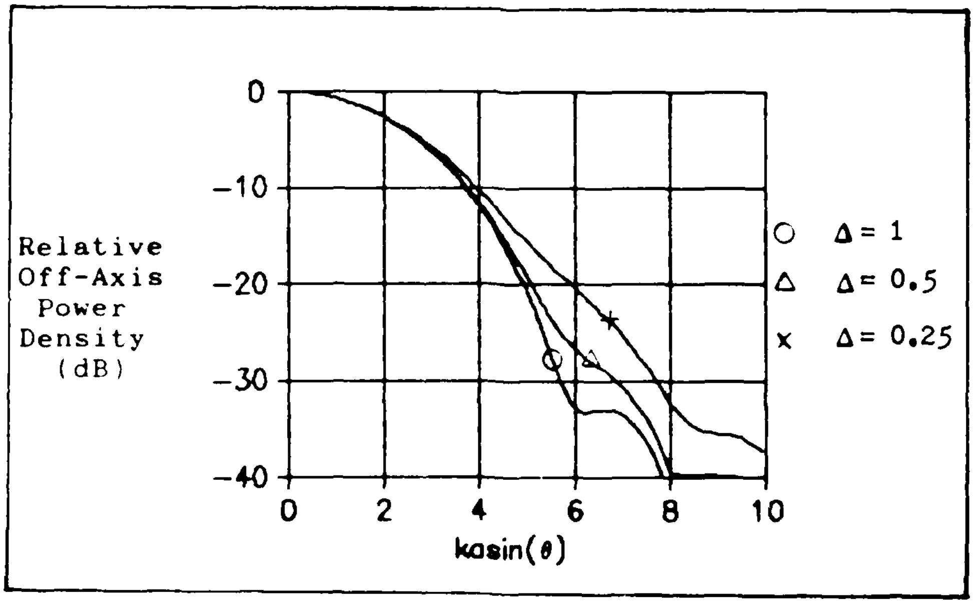

Figure 24. POWERDEN Fresnel Region Radiation Pattern (slr = 35 dB)

VI. Conclusion

Overview

The results presented in the last chapter show that the data produced by POWERDEN in the axial mode corresponds almost exactly with that published by Hansen. The off-axis results cannot be compared directly since none have been published for the one-parameter circular aperture distribution. However, the data produced by POWERDEN in the off-axis mode is consistent in all respects with that which would be expected theoretically.

Recommendations for Further Research

POWERDEN is applicable only to the analysis of circular parabolic reflectors. The method currently used by the Air Force for determining Fresnel region power densities associated with rectangular apertures suffers from the same limitations discussed in Chapter 1 regarding circular apertures. Future research topics might include the development of a better method for the Fresnel region analysis of these rectangular apertures or for phased arrays. Power density calculations for rectangular apertures would require a method for estimating the illumination in each of the principal planes of the aperture.

Summary

The goal of this thesis was the development of a better method for Air Force use in determining the Fresnel region power density hazards associated with paraboloidal antennas. POWERDEN accomplishes that goal; it corrects the three deficiencies of the present method discussed in Chapter 1 by: (1) incorporating a more realistic aperture illumination model, (2) being applicable to a spectrum of specific illuminations instead of the most satisfactory of four predetermined distributions, and (3) unambiguously estimating the specific illumination from the sidelobe ratio instead of the beamwidth. The program is meant to be a useful tool for use by those already familiar with paraboloidal radiation characteristics and the related problems of electromagnetic compatibility.

Appendix A: POWERDEN Source Code

```
cls
print : print
print "      ***************************************"
print "      *                                     *"
print "      *              POWERDEN               *"
print "      *                                     *"
print "      *                                     *"
print "      *                 by                  *"
print "      *                                     *"
print "      *   Johnnie E. Mize, Capt, USAF       *"
print "      *   Air Force Institute of Technology *"
print "      *   Class GE-87D                      *"
print "      *                                     *"
print "      ***************************************"
print
print "    POWERDEN is designed to calculate the Fresnel"
print " region power densities associated with high gain,"
print " center fed, paraboloidal antennas.  It assumes a"
print " diameter-to-wavelength ratio of at least ten."
print " The illumination is modeled as a symmetric taper"
print " on a uniform pedestal with constant phase and"
print " linear polarization."
print : print


'    The aperture illumination model used in this program
'    is the one-parameter circular distribution developed
'    by R. C. Hansen.  Details have been published in:

'       1) IEEE Transactions on Antennas and Propagation,
'           AP-24: 477-480 (July 1976)

'       2) Microwave Journal, 19: 50-52 (February 1976)

'       3) Electronic Letters, 11: 184 (April 1975)

'   The program uses Romberg integration to solve the scalar
'   Kirchoff diffraction integral in the Fresnel Region.

'    A glossary of the main variables is located at the end
'    of this source code.

input "Press <RETURN/ENTER> to proceed. ",ans%
cls

Begin:

input "Input the first sidelobe ratio (17.57 - 95 dB): ",slr
```

```
if slr >= 17.56999 and slr <= 95 then
   print
   row = csrlin
   print "Working ..."
end if

il# = (10^((slr - 17.57)/20))/2        ' from Hansen's formula
pi# = 4*atn(1)
tolerance = 1E-14          ' accuracy of internal calculations

if slr < 17.56999 then     ' choose initial estimate for using
   print "Too small."      ' Newton's method to solve for the
   print                   ' H parameter
   goto Begin
elseif slr < 17.57001 then
   H# = 0
   goto Zero
elseif slr <= 25 then
   est# = 2.9
elseif slr <= 50 then
   est# = 7
elseif slr <= 75 then
   est# = 10.4
elseif slr <= 95 then
   est# = 13.1
else
   print "Too large."
   print
   goto Begin
end if
                           ' calculate the H parameter
do until fnF#(est#) <= tolerance
   est# = est#-fnF#(est#)/fnG#(est#) 'Newton-Raphson formula
loop

H# = est#/pi#

Zero:

locate row,1
print "The H parameter is:";
print using " #.####";H#

Recycle8:

print
input "Input the operating frequency (GHz): ",freq#

if freq# <= 0 then print "Too small." : goto Recycle8

lambda# = 0.30/freq#
k# = 2*pi#/lambda#
print
print "The wavelength is";100*lambda#; "centimeters."
```

```
Recycle1:

print
input "Input the antenna diameter (meters): ",diam#

if diam#<9.99*lambda# then print "Too small." :goto Recycle1

RangeMin# = 0.5*diam#*(diam#/lambda#)^(1/3)
ffmin# = 2*diam#^2/lambda#
a# = diam#/2
print
print "The far field begins at a range of";ffmin#;"meters."

Recycle9:

print
input "Input the antenna gain and_
       transmitted power (dB,kW): ",gain,power

if gain <= 0 or power <= 0 then_
                print "Too small." : goto Recycle9

ffpwrden# = 100*power*(10*(0.1*gain))/_
                      (4*pi#*ffmin#^2)          ' mW/cm^2
print
print "Power density at the beginning of the far field is";
print using " #.######^^^^ ";ffpwrden#;
print "mW/cm^2."

Recycle2:

print
input"Do you need: on-axis(1)  or  off-axis(2)  data? ",ans%

if ans% = 1 then
   exit if
elseif ans% = 2 then
   goto OffAxisStart
else
   goto Recycle2
end if

OnAxisStart:

print
key 10, "0.1" + chr$(13)
print "Input max error for numeric integration";
input " (Default = 0.1%, Press F10): ",ErrLimOn

if ErrLimOn = 0 then print "Too small." : goto OnAxisStart

key 10, chr$(00)
ErrLimOn = ErrLimOn/100
```

```
Recycle10:

print
input "Write data to: line printer(l)_
       or  disc file(f) ? ",answer$

if answer$ = "l" then
   exit if
elseif answer$ = "f" then
   print
   input "Filename:  ",B$
else
   goto Recycle10
end if

print
input "Press <RETURN/ENTER> to proceed. ",ans%
print : print

if DimFlag3% = 1 then DimFlag3% = 0 : erase RealOn#,ImOn#

dim RealOn#(0:195),ImOn#(0:195)'to store integration results
DimFlag3% = 1
flag% = 0 : A$ = "  Re"
u = 0                                  ' so that fnJ#(x#) = 1

FirstDecade:                        ' relative range 1 to 0.1

a% = 100 : b% = 10 : d% = -1 : e% = 10
element% = -1

NextDecade:         ' relative range 0.1 to 0.02, 0.02 to 0.01

e% = 10*e%
for R% = a% to b% step d%
   R = R%/e%
   incr element%

   if DimFlag1% = 1 then DimFlag1% = 0 : erase Romberg#

   dim Romberg#(1:12,1:12)'store intermediate Romberg values
   DimFlag1% = 1
                           ' check for distance from aperture

   if R*ffmin# < RangeMin# then goto StartImOn

   q# = pi#/(8*R)
                                ' compute initial Romberg seed
   if flag% = 0 then
       Romberg#(1,1)=0.25*(cos(q#)+2*fnMidSum#(2))
   else
       Romberg#(1,1)=0.25*(sin(q#)+2*fnMidSum#(2))
   end if
```

```
n% = 2 : j% = 1
print using "####   ";n%;

if Romberg#(1,1) < 0 then
   print using "+#.######^^^^";Romberg#(1,1)
else
   print using " #.######^^^^";Romberg#(1,1)
end if

SubDivideOn:          ' more divisions for greater accuracy

n% = 2*n% : j% = j% + 1 : k% = 1
                         ' exit if no far field convergence
if n% = 8192 and element% = 0 then
   print
   print "No Convergence In The Far Field."
   goto Recycle4
end if
                           ' exit loop after 2^12 divisions
if n% = 8192 and flag% = 0 then
   RealOn#(element%) = 0
   goto Mark0
elseif n% = 8192 and flag% = 1 then
   ImOn#(element%) = 0
   goto Mark0
end if
                         ' compute subsequent Romberg seeds
if flag% = 0 then
    Romberg#(j%,1)=(0.5/n%)*(cos(q#)+2*fnMidSum#(n%))
else
    Romberg#(j%,1)=(0.5/n%)*(sin(q#)+2*fnMidSum#(n%))
end if
                    ' calculate intermediate Romberg values
for q% = (j%-1) to 1 step -1
   k% = k% + 1
   w# = 4^(k%-1)
   Romberg#(q%,k%)=(w#*Romberg#(q%+1,k%-1)_
                   -Romberg#(q%,k%-1))/(w#-1)
next q%
print using "####   ";n%;

if Romberg#(1,k%) < 0 then
   print using "+#.######^^^^";Romberg#(1,k%)
else
   print using " #.######^^^^";Romberg#(1,k%)
end if

er# = 1 - Romberg#(1,k%-1)/Romberg#(1,k%)
                                     ' check for max error
if abs(er#) < ErrLimOn and flag% = 0 then
   RealOn#(element%) = Romberg#(1,k%)
elseif abs(er#) < ErrLimOn and flag% = 1 then
   ImOn#(element%) = Romberg#(1,k%)
```

```
   else
      goto SubDivideOn
   end if

   Mark0:               ' display info about current data point

   print A$;
   print using "          #.####";R;

   if n% = 8192 then
      print "     No Convergence At This Error Limit"
   end if

   print : print : print
   next R%

if element% = 90 then a% = 99 : b% = 20 : d% = -1 : goto_
             NextDecade

if element% = 170 then a% = 196 : b% = 100 : d% = -4 : goto_
             NextDecade

StartImOn:

if flag% = 1 then goto CalculateOn

flag% = 1 : A$ = "  Im" : goto FirstDecade

CalculateOn:                  ' calculate and display axial data

if answer$ = "f" then goto DiscWriteOn

print
print "Job complete ..."
lprint
lprint "                  Axial Power Density Data"
lprint : lprint
lprint "                Relative                        Absolute"
lprint "                 Power          Absolute          Power"
lprint " Relative       Density          Range           Density"
lprint "  Range        (decibels)       (meters)        (mW/cm^2)"
lprint
element% = -1
norm# = (RealOn#(0)^2 + ImOn#(0)^2)    ' normalize wrt ffmin#
a% = 100 : b% = 10 : d% = -1 : e% = 10

Mark5:

e% = 10*e%
for R% = a% to b% step d%
   R = R%/e%

   if R*ffmin# < RangeMin# then goto ParameterListOn
```

```
    incr element%
    s# = (RealOn#(element%)^2+ImOn#(element%)^2)/(R^2*norm#)
    lprint using "        #.####";R;

    if RealOn#(element%) = 0 or ImOn#(element%) = 0 then
       lprint "          *******";
    else
       lprint using "    #######.####";10*log10(s#);
    end if

    lprint using "          #.######^^^^";R*ffmin#;

    if RealOn#(element%) = 0 or ImOn#(element%) = 0 then
       lprint "        ************"
    else
       lprint using "        #.######^^^^";s#*ffpwrden#
    end if
 next R%

 if element% = 90 then a% = 99 :b% = 20 :d% = -1 : goto Mark5

 if element% = 170 then a% = 196 :b%=100 :d% = -4 :goto Mark5

 ParameterListOn:

 lprint
 lprint "       H parameter =";
 lprint using " #.####";H#
 lprint "       sidelobe ratio =";
 lprint using " ##.### ";slr;
 lprint "dB"
 lprint "       frequency =";
 lprint using " #.#####^^^^ ";freq#;
 lprint "GHz"
 lprint "       diameter =";
 lprint using " #.#####^^^^ ";diam#;
 lprint "meters"
 lprint "       gain =";
 lprint using " ##.####^^^^ ";gain;
 lprint "dB"
 lprint "       transmitted power =";
 lprint using " #.######^^^^ ";power;
 lprint "kW"
 lprint "       error limit =";
 lprint using "##.###";100*ErrLimOn;
 lprint "%"
 goto Recycle3

 DiscWriteOn:

 print
 print "Job complete; Writing data to file ";
 print B$
 open B$ for output as #1
```

```
print #1,
print #1,"               Axial Power Density Data"
print #1, : print #1,
print #1,"             Relative                         Absolute"
print #1,"              Power          Absolute          Power"
print #1," Relative    Density          Range           Density"
print #1,"  Range     (decibels)      (meters)       (mW/cm^2)"
print #1,

element% = -1
norm# = (RealOn#(0)^2 + ImOn#(0)^2)    ' normalize wrt ffmin#
a% = 100 : b% = 10 : d% = -1 : e% = 10

f.Mark5:

e% = 10*e%
for R% = a% to b% step d%
   R = R%/e%

   if R*ffmin# < RangeMin# then goto f.ParameterListOn

   incr element%
   s# = (RealOn#(element%)^2+ImOn#(element%)^2)/(R^2*norm#)
   print #1, using "       #.####";R;

   if RealOn#(element%) = 0 or ImOn#(element%) = 0 then
      print #1, "         *******";
   else
      print #1, using "    #######.####";10*log10(s#);
   end if

   print #1, using "         #.######^^^^";R*ffmin#;

   if RealOn#(element%) = 0 or ImOn#(element%) = 0 then
      print #1, "       ************"
   else
      print #1, using "       #.######^^^^";s#*ffpwrden#
   end if

next R%

if element% = 90 then a% = 99 :b%=20 :d% = -1 :goto f.Mark5

if element% = 170 then a%=196:b%=100:d%=-4:goto f.Mark5

f.ParameterListOn:

print #1,
print #1, "       H parameter =";
print #1, using " #.####";H#
print #1, "       sidelobe ratio =";
print #1, using " ##.### ";slr;
print #1, "dB"
print #1, "       frequency =";
```

```
print #1, using " #.#####^^^^ ";freq#;
print #1, "GHz"
print #1, "       diameter =";
print #1, using " #.#####^^^^ ";diam#;
print #1, "meters"
print #1, "       gain =";
print #1, using " ##.####^^^^ ";gain;
print #1, "dB"
print #1, "       transmitted power =";
print #1, using " #.######^^^^ ";power;
print #1, "kW"
print #1, "       error limit =";
print #1, using "##.###";100*ErrLimOn;
print #1, "%"
close #1

Recycle3:

print
input "Do you need off-axis data using_
       the current parameters (y/n)? ",ans$

if ans$ = "y" then
   goto OffAxisStart
elseif ans$ = "n" then
   exit if
else
   goto Recycle3
end if

Recycle4:

print
input "New problem (y/n)? ",ans$

if ans$ = "y" then
   cls
   goto Begin
elseif ans$ = "n" then
   exit if
else
   goto Recycle4
end if
end

OffAxisStart:

print
input "Start off-axis calculations at what_
       axial range (meters)? ",z#

if z# < RangeMin# then
   print "Too close."
   goto OffAxisStart
```

```
elseif z# > ffmin# + 0.001 then
   print "That's in the far field."
   goto OffAxisStart
end if
                                    ' compute max angle off-axis

SinSquareThetaMax# = (0.08*lambda#*z#)/(pi#*a#^2)
ThetaMax# = atn(sqr(SinSquareThetaMax#/_
            (1 - SinSquareThetaMax#)))

Recycle7:

key 10, "16" + chr$(13)
print
print "Input a value (Default = 16, Press F10) for_
       the range of kasin(";
print chr$(233);
input "): ",f

if abs(f) > k#*a#*sin(ThetaMax#) then
   print "Too large; max =";
   print k#*a#*sin(ThetaMax#)
   goto Recycle7
else
   f% = fix(10*f)
   key 10, chr$(00)
end if

Recycle12:

print
key 10, "0.1" + chr$(13)
print "Input max error for numeric integration";
input " (Default = 0.1%, Press F10): ",ErrLimOff

if ErrLimOff = 0 then print "Too small." : goto Recycle12

key 10, chr$(00)
ErrLimOff = ErrLimOff/100

Recycle11:

print
input "Write data to: line printer(l)_
       or  disc file(f) ? ",answer$

if answer$ = "l" then
   exit if
elseif answer$ = "f" then
   print
   input "Filename:  ",B$
else
   goto Recycle11
end if
```

```
print
input "Press <RETURN/ENTER> to proceed. ",ans%
print
flag% = 0 : A$ = "  Re"

if DimFlag2% = 1 then DimFlag2% = 0 : erase RealOff#,ImOff#
                              'to store integration results
dim RealOff#(0:f%),ImOff#(0:f%)
DimFlag2% = 1

Mark1:

NormFlag% = 0
print
row = csrlin - 1
print "Working ..."

Mark2:

element% = -1
for u% = 0 to f%                       ' range of kasin(theta)
   u = u%/10
   incr element%

   if DimFlag1% = 1 then DimFlag1% = 0 : erase Romberg#

   dim Romberg#(1:12,1:12)'store intermediate Romberg values
   DimFlag1% = 1
   SinSquareTheta# = (u/(k#*a#))^2
   Theta# = atn(sqr(SinSquareTheta#/(1-SinSquareTheta#)))

   if NormFlag% = 0 then R# = 1_
                    else R# = z#/(ffmin#*cos(Theta#))

   q# = pi#/(8*R#)
                              ' compute initial Romberg seed
   if flag% = 0 then
      Romberg#(1,1) = 0.25*(cos(q#)*fnJ#(u)+2*fnMidSum#(2))
   else
      Romberg#(1,1) = 0.25*(sin(q#)*fnJ#(u)+2*fnMidSum#(2))
   end if

   n% = 2 : j% = 1

   if NormFlag% = 1 and element% = 0 then locate row,1

   if NormFlag% = 1 then
      print using "####   ";n%;
      if Romberg#(1,1) < 0 then
         print using "+#.######^^^^";Romberg#(1,1)
      else
         print using " #.######^^^^";Romberg#(1,1)
      end if
   end if
```

```
SubDivideOff:        ' more divisions for greater accuracy

n% = 2*n% : j% = j% + 1 : k% = 1
                            ' exit if no far field convergence

if n% = 8192 and NormFlag% = 0 then
   locate row,1
   print "No Convergence In The Far Field."
   goto Recycle6
end if

if n% = 8192 and flag% = 0 then
   RealOff#(element%) = 0 'exit loop after 2^12 divisions
   goto Mark3
elseif n% = 8192 and flag% = 1 then
   ImOff#(element%) = 0
   goto Mark3
end if
                            ' compute subsequent Romberg seeds
if flag% = 0 then
   Romberg#(j%,1) = (0.5/n%)*_
                    (cos(q#)*fnJ#(u)+2*fnMidSum#(n%))
else
   Romberg#(j%,1) = (0.5/n%)*_
                    (sin(q#)*fnJ#(u)+2*fnMidSum#(n%))
end if
                       ' calculate intermediate Romberg values
for q% = (j%-1) to 1 step -1
   k% = k% + 1
   w# = 4^(k%-1)
   Romberg#(q%,k%)=(w#*Romberg#(q%+1,k%-1)_
                   -Romberg#(q%,k%-1))/(w#-1)
next q%

if NormFlag% = 1 then
   print using "####    ";n%;
   if Romberg#(1,k%) < 0 then
      print using "+#.######^^^^";Romberg#(1,k%)
   else
      print using " #.######^^^^";Romberg#(1,k%)
   end if
end if

er# = 1 - Romberg#(1,k%-1)/Romberg#(1,k%)

                                        ' check for max error
if abs(er#) < ErrLimOff and flag% = 0 then
   RealOff#(element%) = Romberg#(1,k%)
elseif abs(er#) < ErrLimOff and flag% = 1 then
   ImOff#(element%) = Romberg#(1,k%)
else
   goto SubDivideOff
end if
```

```
   Mark3:              ' display info about current data point

   if NormFlag% = 1 then
      print A$;
      print using "         #####.#";u;
      if n% = 8192 then
         print "     No Convergence At This Error Limit"
      end if
      print : print : print
   end if
                                       ' store normalization data
   if NormFlag% = 0 and flag% = 0 then
      HoldRealNorm# = RealOff#(0)
      NormFlag% = 1
      goto Mark2
   elseif NormFlag% = 0 and flag% = 1 then
      HoldImNorm# = ImOff#(0)
      NormFlag% = 1
      goto Mark2
   end if

next u%

StartImOff:

if flag% = 1 then goto CalculateOff

flag% = 1 : A$ = "  Im" : goto Mark1

CalculateOff:          ' calculate and display off-axis data

if answer$ = "f" then goto DiscWriteOff

print
print "Job complete ..."
lprint
lprint "              Off-Axis Power Density Data"
lprint : lprint
lprint "                Relative                    Absolute"
lprint "                 Power        Distance       Power"
lprint "                Density       Off-Axis      Density"
lprint "  kasin(";
lprint chr$(233);
lprint ")             (decibels)     (meters)     (mW/cm^2)"
lprint

element% = -1                           ' normalize wrt ffmin#
norm# = 4*(HoldRealNorm#^2 + HoldImNorm#^2)

for u% = 0 to f%                        ' range of kasin(theta)
   u = u%/10
   incr element%
   SinSquareTheta# = (u/(k#*a#))^2
```

```
    Theta# = atn(sqr(SinSquareTheta#/(1-SinSquareTheta#)))

    R# = z#/(ffmin#*cos(Theta#))    ' calculate relative range

    if element% = 0 then g# = 0 else g# = z#*tan(Theta#)

    s#=((1+cos(Theta#))/R#)^2_
            *(RealOff#(element%)^2+ImOff#(element%)^2)/norm#

    if element% = 0 then save# = s#

    lprint using "     #####.#     ";u;

    if RealOff#(0) = 0 or ImOff#(0) = 0 then
       lprint "       ********     ";
    elseif RealOff#(element%)= 0 or ImOff#(element%) = 0 then
       lprint "       ********     ";
    else
       lprint using "  #######.####     ";10*log10(s#/save#);
    end if

    lprint using "      #.######^^^^   ";g#;

    if RealOff#(element%) = 0 or ImOff#(element%) = 0 then
       lprint "     ************"
    else
       lprint using "      #.######^^^^";s#*ffpwrden#
    end if

next u%

ParameterListOff:

lprint
lprint "       H parameter =";
lprint using " #.####";H#
lprint "       sidelobe ratio =";
lprint using " ##.### ";slr;
lprint "dB"
lprint "       frequency =";
lprint using " #.#####^^^^ ";freq#;
lprint "GHz"
lprint "       diameter =";
lprint using " #.#####^^^^ ";diam#;
lprint "meters"
lprint "       gain =";
lprint using " ##.####^^^^ ";gain;
lprint "dB"
lprint "       transmitted power = ";
lprint using " #.######^^^^ ";power;
lprint "kW"
lprint "       off-axis starting point =";
lprint using " #.#######^^^^ ";z#;
lprint "meters"
```

```
lprint "        error limit =";
lprint using "##.###";100*ErrLimOff;
lprint "%"
goto Recycle5

DiscWriteOff:

print
print "Job complete; writing data to file ";
print B$
open B$ for output as #2
print #2,
print #2,                   Off-Axis Power Density Data"
print #2, : print #2,
print #2,"                    Relative                      Absolute"
print #2,"                     Power       Distance          Power"
print #2,"                    Density      Off-Axis         Density"
print #2,"kasin(";
print #2, chr$(233);
print #2, ")               (decibels)    (meters)         (mW/cm^2)"
print #2,

element% = -1                              ' normalize wrt ffmin#
norm# = 4*(HoldRealNorm#^2 + HoldImNorm#^2)

for u% = 0 to f%                           ' range of kasin(theta)
   u = u%/10
   incr element%
   SinSquareTheta# = (u/(k#*a#))^2
   Theta# = atn(sqr(SinSquareTheta#/(1-SinSquareTheta#)))

   R# = z#/(ffmin#*cos(Theta#))    ' calculate relative range

   if element% = 0 then g# = 0 else g# = z#*tan(Theta#)

   s#=((1+cos(Theta#))/R#)^2_
        *(RealOff#(element%)^2+ImOff#(element%)^2)/norm#

   if element% = 0 then save# = s#

   print #2, using "    #####.#    ";u;

   if RealOff#(0) = 0 or ImOff#(0) = 0 then
      print #2, "      ********    ";
   elseif RealOff#(element%) =0 or ImOff#(element%) = 0 then
      print #2, "      ********    ";
   else
      print #2, using "#######.####     ";10*log10(s#/save#);
   end if

   print #2, using "     #.######^^^^  ";g#;
```

```
   if RealOff#(element%) = 0 or ImOff#(element%) = 0 then
      print #2, "    ************"
   else
      print #2, using "     #.######^^^^";s#*ffpwrden#
   end if

next u%

f.ParameterListOff:

print #2,
print #2, "       H parameter =";
print #2, using " #.####";H#
print #2, "       sidelobe ratio =";
print #2, using " ##.### ";slr;
print #2, "dB"
print #2, "       frequency =";
print #2, using " #.#####^^^^ ";freq#;
print #2, "GHz"
print #2, "       diameter =";
print #2, using " #.#####^^^^ ";diam#;
print #2, "meters"
print #2, "       gain =";
print #2, using " ##.####^^^^ ";gain;
print #2, "dB"
print #2, "       transmitted power =";
print #2, using " #.######^^^^ ";power;
print #2, "kW"
print #2, "       off-axis starting point =";
print #2, using " #.#######^^^^ ";z#;
print #2, "meters"
print #2, "       error limit =";
print #2, using "##.###";100*ErrLimOff;
print #2, "%"
close #2

Recycle5:

print
print "Do you need more off-axis data ";
input "using the current parameters (y/n)? ",ans$

if ans$ = "y" then
   goto OffAxisStart
elseif ans$ = "n" then
   exit if
else
   goto Recycle5
end if

Recycle6:

print
input "New problem (y/n)? ",ans$
```

```
if ans$ = "y" then
   cls
   goto Begin
elseif ans$ = "n" then
   exit if
else
   goto Recycle6
end if
end

def fnF#(x#)                  ' used in Newton-Raphson formula
   local i%,w#,j#             ' for finding the H parameter
   w# = 0 : j# = 1E06
   for i% = 1 to 170
      w#=w# + x#^(2*i%)/(2^(2*i%+1)*fnFac#(i%)*fnFac#(i%+1))
      if abs(w#-j#) < tolerance then exit for else j# = w#
   next i%
   fnF# = w# + 0.5 - i1#
end def

def fnG#(x#)                  ' returns derivative of fnF#(x#)
   local i%,y#,j#
   y# = 0 : j# = 1E06
   for i% = 1 to 170
      y# = y#+2*i%*x#^(2*i%-1)/(2^(2*i%+1)_
                 *fnFac#(i%)*fnFac#(i%+1))
      if abs(y#-j#) < tolerance then exit for else j# = y#
   next i%
   fnG# = y#
end def

def fnFac#(x%)              ' returns the factorial of a number
   local i%,e#
   e# = 1
   for i% = 1 to x%
      e# = e# * i%
   next i%
   fnFac# = e#
end def
                           ' returns sum of midpoint evaluations
def fnMidSum#(x%)          ' when using Trapezoidal Rule during
   local inc#,p#,i%,b#,d#       ' Romberg integration
   inc# = 1/x% : b# = 0
   for i% = 1 to (x%-1)
      p# = 1-i%*inc#
      if flag% = 0 then
         d# = cos(q#*p#^2)
      else
         d# = sin(q#*p#^2)
      end if
      b# = b# + d#*p#*fnI#(pi#*H#*sqr(1-p#^2))*fnJ#(u*p#)
   next i%
   fnMidSum# = b#
end def
```

```
def fnI#(x#)                    ' returns modified Bessel function
   local i%,v#,j#               ' of the first kind, order zero
   v# = 0 : j# = 1E06
   for i% = 1 to 170
      v# = v# + (x#/2)^(2*i%)/fnFac#(i%)^2
      if abs(v#-j#) < tolerance then exit for else j# = v#
   next i%
   fnI# = v# + 1
end def

     ' returns Bessel function of the first kind, order zero
def fnJ#(x#)
   if x# > 25 then fnJ# = fnA#(x#)_
                       *sin(x#)+fnB#(x#)*cos(x#) : exit def
   local i%,c#,m#
   c# = 0 : m# = 1E06
   for i% = 1 to 170
      c# = c# + (-1)^i%*(x#/2)^(2*i%)/fnFac#(i%)^2
      if abs(c#-m#) < tolerance then exit for else m# = c#
   next i%
   fnJ# = c# + 1
end def
        ' fnA#,fnB#,fnP#,fnQ# used when x# > 25 for fnJ#(x#)

def fnA#(x#) = (fnP#(x#) - fnQ#(x#))/sqr(pi#*x#)

def fnB#(x#) = (fnP#(x#) + fnQ#(x#))/sqr(pi#*x#)

def fnP#(x#)
   local w%,c%,check#,m%,sum#,i%,product#
   m% = 0 : w% = -1 : c% = 0 : check# = 1E06 : sum# = 0
   RoutineP:
   w% = w% + 4 : c% = c% + 2 : m% = m% + 1 : product# = 1
   for i% = 1 to w% step 2
      product# = product# * i%^2
   next i%
   product# = product#*(-1)^m%
   product# = product#/fnFac#(c%)
   product# = product#/(8*x#)^c%
   sum# = sum# + product#
   if abs(sum# - check#) < tolerance then
      sum# = sum# + 1
   else
      check# = sum#
      goto RoutineP
   end if
   fnP# = sum#
end def
```

```
def fnQ#(x#)
   local w%,c%,check#,m%,sum#,i%,product#
   m% = 0 : w% = -3 : c% = -1 : check# = 1E06 : sum# = 0
   RoutineQ:
   w% = w% + 4 : c% = c% + 2 : m% = m% + 1 : product# = 1
   for i% = 1 to w% step 2
      product# = product# * i%^2
   next i%
   product# = product#*(-1)^m%
   product# = product#/fnFac#(c%)
   product# = product#/(8*x#)^c%
   sum# = sum# + product#

   if abs(sum# - check#) < tolerance then
      exit if
   else
      check# = sum#
      goto RoutineQ
   end if
   fnQ# = sum#
end def


'                         **********
'                          GLOSSARY
'                         **********

' slr ............. antenna sidelobe ratio

' H# .............. H parameter for one-parameter
'                   distribution developed by Hansen

' pi# ............. 3.1416

' est# ............ iterative value of pi#*H# used in loop
'                   employing Newton's method to solve for
'                   the H parameter

' tolerance ....... accuracy to which internal calculations
'                   are computed

' freq# ........... operating frequency

' lambda# ......... wavelength

' k# .............. propagation constant

' diam# ........... antenna diameter

' RangeMin# ....... minimum axial range

' ffmin# .......... minimum far field distance
```

```
' a# .............. antenna radius

' gain ............ antenna power gain in decibels

' power ........... transmitted power (peak or average
'                   depending on the hazard criteria)

' ffpwrden# ....... power density at ffmin#

' ErrLimOn ........ maximum error accepted by Romberg
'                   integration routine when calculating
'                   axial power density

' RealOn# ......... linear array element; stores real part
'                   of integral during axial computations

' ImOn# ........... linear array element; stores imaginary
'                   part of integral during axial
'                   computations

' u ............... k#*a#*sin(theta)

' R,R# ............ relative distance from aperture origin
'                   to field point; normalized with respect
'                   to ffmin#

' Romberg# ........ 12x12 array element; stores all elements
'                   of the Romberg matrix until the error
'                   limit is reached

' z# .............. axial range at which off-axis
'                   computations begin

' ThetaMax# ....... maximum off-axis angle

' Theta# .......... present off-axis angle

' f ............... value of k#*a#*sin(Theta#) to preceed to
'                   off-axis

' ErrLimOff ....... maximum error accepted by Romberg
'                   integration routine when calculating
'                   off-axis power density

' RealOff# ........ linear array element; store real part of
'                   integral during off-axis computations

' ImOff# .......... linear array element; store imaginary
'                   part of integral during off-axis
'                   computations
```

```
' HoldRealNorm# ... stores real part of integral at the far
'                   field distance for normalization
'                   purposes during off-axis computations

' HoldImNorm# ..... stores imaginary part of integral at the
'                   far field distance for normalization
'                   purposes during off-axis computations
```

Appendix B: POWERDEN Axial Output Data

Axial Power Density Data

Relative Range	Relative Power Density (decibels)	Absolute Range (meters)	Absolute Power Density (mW/cm^2)
1.0000	0.0000	2.400000E+03	1.000000E+01
0.9900	0.0862	2.376000E+03	1.020037E+01
0.9800	0.1732	2.352000E+03	1.040680E+01
0.9700	0.2611	2.328000E+03	1.061952E+01
0.9600	0.3498	2.304000E+03	1.083881E+01
0.9500	0.4395	2.280000E+03	1.106492E+01
0.9400	0.5301	2.256000E+03	1.129814E+01
0.9300	0.6216	2.232000E+03	1.153878E+01
0.9200	0.7141	2.208000E+03	1.178715E+01
0.9100	0.8076	2.184000E+03	1.204357E+01
0.9000	0.9020	2.160000E+03	1.230842E+01
0.8900	0.9975	2.136000E+03	1.258204E+01
0.8800	1.0940	2.112000E+03	1.286483E+01
0.8700	1.1916	2.088000E+03	1.315721E+01
0.8600	1.2903	2.064000E+03	1.345960E+01
0.8500	1.3901	2.040000E+03	1.377248E+01
0.8400	1.4911	2.016000E+03	1.409632E+01
0.8300	1.5932	1.992000E+03	1.443164E+01
0.8200	1.6964	1.968000E+03	1.477898E+01
0.8100	1.8010	1.944000E+03	1.513893E+01
0.8000	1.9067	1.920000E+03	1.551209E+01
0.7900	2.0137	1.896000E+03	1.589912E+01
0.7800	2.1221	1.872000E+03	1.630071E+01
0.7700	2.2317	1.848000E+03	1.671758E+01
0.7600	2.3428	1.824000E+03	1.715052E+01
0.7500	2.4552	1.800000E+03	1.760035E+01
0.7400	2.5691	1.776000E+03	1.806796E+01
0.7300	2.6844	1.752000E+03	1.855429E+01
0.7200	2.8013	1.728000E+03	1.906034E+01
0.7100	2.9197	1.704000E+03	1.958719E+01
0.7000	3.0397	1.680000E+03	2.013597E+01
0.6900	3.1614	1.656000E+03	2.070790E+01
0.6800	3.2847	1.632000E+03	2.130430E+01
0.6700	3.4097	1.608000E+03	2.192656E+01
0.6600	3.5365	1.584000E+03	2.257618E+01
0.6500	3.6651	1.560000E+03	2.325477E+01
0.6400	3.7956	1.536000E+03	2.396406E+01
0.6300	3.9280	1.512000E+03	2.470589E+01
0.6200	4.0624	1.488000E+03	2.548226E+01
0.6100	4.1988	1.464000E+03	2.629532E+01
0.6000	4.3373	1.440000E+03	2.714739E+01
0.5900	4.4779	1.416000E+03	2.804096E+01

0.5800	4.6208	1.392000E+03	2.897871E+01
0.5700	4.7659	1.368000E+03	2.996357E+01
0.5600	4.9134	1.344000E+03	3.099869E+01
0.5500	5.0634	1.320000E+03	3.208747E+01
0.5400	5.2158	1.296000E+03	3.323363E+01
0.5300	5.3708	1.272000E+03	3.444118E+01
0.5200	5.5284	1.248000E+03	3.571449E+01
0.5100	5.6889	1.224000E+03	3.705830E+01
0.5000	5.8521	1.200000E+03	3.847780E+01
0.4900	6.0183	1.176000E+03	3.997863E+01
0.4800	6.1875	1.152000E+03	4.156697E+01
0.4700	6.3598	1.128000E+03	4.324954E+01
0.4600	6.5354	1.104000E+03	4.503373E+01
0.4500	6.7143	1.080000E+03	4.692762E+01
0.4400	6.8966	1.056000E+03	4.894008E+01
0.4300	7.0826	1.032000E+03	5.108086E+01
0.4200	7.2722	1.008000E+03	5.336068E+01
0.4100	7.4657	9.840000E+02	5.579131E+01
0.4000	7.6631	9.600000E+02	5.838575E+01
0.3900	7.8646	9.360000E+02	6.115836E+01
0.3800	8.0703	9.120000E+02	6.412492E+01
0.3700	8.2803	8.880000E+02	6.730292E+01
0.3600	8.4949	8.640000E+02	7.071166E+01
0.3500	8.7141	8.400000E+02	7.437247E+01
0.3400	8.9381	8.160000E+02	7.830890E+01
0.3300	9.1670	7.920000E+02	8.254696E+01
0.3200	9.4009	7.680000E+02	8.711535E+01
0.3100	9.6400	7.440000E+02	9.204558E+01
0.3000	9.8844	7.200000E+02	9.737226E+01
0.2900	10.1340	6.960000E+02	1.031331E+02
0.2800	10.3889	6.720000E+02	1.093690E+02
0.2700	10.6492	6.480000E+02	1.161238E+02
0.2600	10.9147	6.240000E+02	1.234437E+02
0.2500	11.1852	6.000000E+02	1.313763E+02
0.2400	11.4601	5.760000E+02	1.399627E+02
0.2300	11.7394	5.520000E+02	1.492590E+02
0.2200	12.0220	5.280000E+02	1.592955E+02
0.2100	12.3069	5.040000E+02	1.700928E+02
0.2000	12.5922	4.800000E+02	1.816440E+02
0.1900	12.8757	4.560000E+02	1.938961E+02
0.1800	13.1539	4.320000E+02	2.067216E+02
0.1700	13.4217	4.080000E+02	2.198743E+02
0.1600	13.6721	3.840000E+02	2.329211E+02
0.1500	13.8941	3.600000E+02	2.451403E+02
0.1400	14.0717	3.360000E+02	2.553681E+02
0.1300	14.1794	3.120000E+02	2.617814E+02
0.1200	14.1766	2.880000E+02	2.616136E+02
0.1100	13.9944	2.640000E+02	2.508631E+02
0.1000	13.5074	2.400000E+02	2.242541E+02
0.0990	13.4340	2.376000E+02	2.204970E+02
0.0980	13.3550	2.352000E+02	2.165187E+02
0.0970	13.2698	2.328000E+02	2.123147E+02
0.0960	13.1781	2.304000E+02	2.078806E+02
0.0950	13.0795	2.280000E+02	2.032129E+02

0.0940	12.9734	2.256000E+02	1.983086E+02
0.0930	12.8595	2.232000E+02	1.931761E+02
0.0920	12.7368	2.208000E+02	1.877923E+02
0.0910	12.6047	2.184000E+02	1.821674E+02
0.0900	12.4626	2.160000E+02	1.763022E+02
0.0890	12.3096	2.136000E+02	1.701984E+02
0.0880	12.1447	2.112000E+02	1.638596E+02
0.0870	11.9670	2.088000E+02	1.572906E+02
0.0860	11.7753	2.064000E+02	1.504985E+02
0.0850	11.5683	2.040000E+02	1.434921E+02
0.0840	11.3444	2.016000E+02	1.362829E+02
0.0830	11.1020	1.992000E+02	1.288847E+02
0.0820	10.8391	1.968000E+02	1.213145E+02
0.0810	10.5535	1.944000E+02	1.135922E+02
0.0800	10.2425	1.920000E+02	1.057416E+02
0.0790	9.9030	1.896000E+02	9.779032E+01
0.0780	9.5313	1.872000E+02	8.976996E+01
0.0770	9.1231	1.848000E+02	8.171710E+01
0.0760	8.6731	1.824000E+02	7.367328E+01
0.0750	8.1747	1.800000E+02	6.568555E+01
0.0740	7.6198	1.776000E+02	5.780662E+01
0.0730	6.9980	1.752000E+02	5.009560E+01
0.0720	6.2959	1.728000E+02	4.261813E+01
0.0710	5.4958	1.704000E+02	3.544680E+01
0.0700	4.5729	1.680000E+02	2.866114E+01
0.0690	3.4924	1.656000E+02	2.234827E+01
0.0680	2.2017	1.632000E+02	1.660233E+01
0.0670	0.6165	1.608000E+02	1.152522E+01
0.0660	-1.4115	1.584000E+02	7.225278E+00
0.0650	-4.1823	1.560000E+02	3.817386E+00
0.0640	-8.4713	1.536000E+02	1.421895E+00
0.0630	-17.8700	1.512000E+02	1.633047E-01
0.0620	-17.7311	1.488000E+02	1.686141E-01
0.0610	-8.0551	1.464000E+02	1.564900E+00
0.0600	-3.4908	1.440000E+02	4.476353E+00
0.0590	-0.4477	1.416000E+02	9.020411E+00
0.0580	1.8476	1.392000E+02	1.530248E+01
0.0570	3.6944	1.368000E+02	2.341234E+01
0.0560	5.2393	1.344000E+02	3.341413E+01
0.0550	6.5648	1.320000E+02	4.534000E+01
0.0540	7.7218	1.296000E+02	5.918129E+01
0.0530	8.7432	1.272000E+02	7.487274E+01
0.0520	9.6514	1.248000E+02	9.228749E+01
0.0510	10.4618	1.224000E+02	1.112195E+02
0.0500	11.1850	1.200000E+02	1.313707E+02
0.0490	11.8281	1.176000E+02	1.523375E+02
0.0480	12.3955	1.152000E+02	1.735984E+02
0.0470	12.8893	1.128000E+02	1.945051E+02
0.0460	13.3098	1.104000E+02	2.142772E+02
0.0450	13.6547	1.080000E+02	2.319897E+02
0.0440	13.9209	1.056000E+02	2.466537E+02
0.0430	14.1022	1.032000E+02	2.571714E+02
0.0420	14.1892	1.008000E+02	2.623741E+02
0.0410	14.1698	9.840000E+01	2.612019E+02

0.0400	14.0268	9.600000E+01	2.527415E+02
0.0390	13.7364	9.360000E+01	2.363942E+02
0.0380	13.2646	9.120000E+01	2.120604E+02
0.0370	12.5612	8.880000E+01	1.803533E+02
0.0360	11.5466	8.640000E+01	1.427769E+02
0.0350	10.0919	8.400000E+01	1.021381E+02
0.0340	7.9388	8.160000E+01	6.221326E+01
0.0330	4.4881	7.920000E+01	2.810652E+01
0.0320	-2.4743	7.680000E+01	5.656799E+00
0.0310	-11.7133	7.440000E+01	6.740233E-01
0.0300	2.4552	7.200000E+01	1.760035E+01
0.0290	7.6077	6.960000E+01	5.764668E+01
0.0280	10.6693	6.720000E+01	1.166618E+02
0.0270	12.6341	6.480000E+01	1.834039E+02
0.0260	13.7928	6.240000E+01	2.394866E+02
0.0250	14.1953	6.000000E+01	2.627413E+02
0.0240	13.7216	5.760000E+01	2.355940E+02
0.0230	11.9893	5.520000E+01	1.580988E+02
0.0220	7.8065	5.280000E+01	6.034628E+01
0.0210	-8.3348	5.040000E+01	1.467297E+00
0.0200	5.8521	4.800000E+01	3.847767E+01
0.0196	9.1419	4.704000E+01	8.207168E+01
0.0192	11.3248	4.608000E+01	1.356690E+02
0.0188	12.8017	4.512000E+01	1.906204E+02
0.0184	13.7290	4.416000E+01	2.359914E+02
0.0180	14.1622	4.320000E+01	2.607458E+02
0.0176	14.0827	4.224000E+01	2.560185E+02
0.0172	13.4052	4.128000E+01	2.190368E+02
0.0168	11.9255	4.032000E+01	1.557942E+02
0.0164	9.1520	3.936000E+01	8.226215E+01
0.0160	3.4518	3.840000E+01	2.214004E+01
0.0156	-19.7247	3.744000E+01	1.065454E-01
0.0152	4.9309	3.648000E+01	3.112377E+01
0.0148	10.3810	3.552000E+01	1.091689E+02
0.0144	13.0526	3.456000E+01	2.019570E+02
0.0140	14.1405	3.360000E+01	2.594476E+02
0.0136	13.7976	3.264000E+01	2.397512E+02
0.0132	11.5793	3.168000E+01	1.438582E+02
0.0128	5.3188	3.072000E+01	3.403135E+01
0.0124	-3.7740	2.976000E+01	4.193701E+00
0.0120	9.8842	2.880000E+01	9.736940E+01
0.0116	13.6454	2.784000E+01	2.314964E+02
0.0112	13.9155	2.688000E+01	2.463492E+02
0.0108	10.0464	2.592000E+01	1.010746E+02
0.0104	-16.2037	2.496000E+01	2.396779E-01
0.0100	11.1850	2.400000E+01	1.313708E+02

H parameter = 0.0000
sidelobe ratio = 17.570 dB
frequency = 1.00000E+01 GHz
diameter = 6.00000E+00 meters
gain = 55.5860E+00 dB
transmitted power = 2.000000E+01 kW
error limit = 0.100%

Appendix C: POWERDEN Off-Axis Output Data

Off-Axis Power Density Data

kasin(θ)	Relative Power Density (decibels)	Distance Off-Axis (meters)	Absolute Power Density (mW/cm^2)
0.0	0.0000	0.000000E+00	3.847780E+01
0.1	-0.0109	1.909859E-01	3.838170E+01
0.2	-0.0435	3.819719E-01	3.809463E+01
0.3	-0.0979	5.729579E-01	3.762018E+01
0.4	-0.1743	7.639439E-01	3.696430E+01
0.5	-0.2728	9.549300E-01	3.613515E+01
0.6	-0.3937	1.145916E+00	3.514299E+01
0.7	-0.5373	1.336902E+00	3.399998E+01
0.8	-0.7040	1.527889E+00	3.271997E+01
0.9	-0.8941	1.718875E+00	3.131830E+01
1.0	-1.1083	1.909862E+00	2.981148E+01
1.1	-1.3470	2.100849E+00	2.821696E+01
1.2	-1.6110	2.291835E+00	2.655280E+01
1.3	-1.9010	2.482822E+00	2.483743E+01
1.4	-2.2180	2.673810E+00	2.308931E+01
1.5	-2.5629	2.864797E+00	2.132665E+01
1.6	-2.9368	3.055785E+00	1.956714E+01
1.7	-3.3412	3.246773E+00	1.782768E+01
1.8	-3.7773	3.437761E+00	1.612422E+01
1.9	-4.2470	3.628749E+00	1.447135E+01
2.0	-4.7522	3.819738E+00	1.288232E+01
2.1	-5.2950	4.010727E+00	1.136878E+01
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2.3	-6.5040	4.392706E+00	8.606241E+00
2.4	-7.1764	4.583696E+00	7.371804E+00
2.5	-7.8989	4.774686E+00	6.241969E+00
2.6	-8.6759	4.965677E+00	5.219358E+00
2.7	-9.5124	5.156668E+00	4.304985E+00
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2.9	-11.3855	5.538651E+00	2.796808E+00
3.0	-12.4336	5.729643E+00	2.197087E+00
3.1	-13.5625	5.920636E+00	1.694163E+00
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6.3	-22.8189	1.203272E+01	2.010560E-01
6.4	-24.0326	1.222373E+01	1.520388E-01
6.5	-25.4298	1.241475E+01	1.102111E-01
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9.3	-26.8239	1.776364E+01	7.994962E-02
9.4	-27.7467	1.795469E+01	6.464507E-02
9.5	-28.8305	1.814574E+01	5.036850E-02

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10.3	-39.8928	1.967420E+01	3.943945E-03
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10.9	-30.0728	2.082060E+01	3.783777E-02
11.0	-29.3546	2.101167E+01	4.464244E-02
11.1	-28.7824	2.120275E+01	5.092936E-02
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11.3	-28.0014	2.158490E+01	6.096320E-02
11.4	-27.7677	2.177598E+01	6.433388E-02
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14.5	-31.1393	2.770034E+01	2.959911E-02
14.6	-30.9229	2.789148E+01	3.111110E-02
14.7	-30.7991	2.808262E+01	3.201101E-02
14.8	-30.7635	2.827376E+01	3.227482E-02
14.9	-30.8136	2.846491E+01	3.190443E-02

15.0	-30.9488	2.865606E+01	3.092651E-02
15.1	-31.1700	2.884721E+01	2.939050E-02
15.2	-31.4800	2.903836E+01	2.736584E-02
15.3	-31.8834	2.922952E+01	2.493841E-02
15.4	-32.3873	2.942067E+01	2.220671E-02
15.5	-33.0016	2.961183E+01	1.927743E-02
15.6	-33.7406	2.980299E+01	1.626117E-02
15.7	-34.6241	2.999416E+01	1.326797E-02
15.8	-35.6805	3.018532E+01	1.040297E-02
15.9	-36.9519	3.037649E+01	7.762789E-03
16.0	-38.5026	3.056766E+01	5.431861E-03

H parameter = 0.0000
sidelobe ratio = 17.570 dB
frequency = 1.00000E+01 GHz
diameter = 6.00000E+00 meters
gain = 55.5860E+00 dB
transmitted power = 2.000000E+01 kW
off-axis starting point = 1.2000000E+03 meters
error limit = 0.100%

Appendix D: Derivation of Eq (15)

The scalar Kirchoff diffraction integral with the FSA approximations is

$$E_P = B(\theta)\int_0^{2\pi}\int_0^{a} G(\rho)\exp\left[\frac{-jk\rho^2}{2R}\right]\exp[jk\rho\sin(\theta)\cos(\phi-\phi')]\rho d\rho d\phi' \qquad (6)$$

where

$$B(\theta) = (1+\cos(\theta))\ (jk/4\pi R)\ \exp(-jkR)$$

and all other terms are as defined originally in Chapter 2. Since only relative amplitude is desired, the constant jk and the phase term can be dropped from $B(\theta)$ resulting in

$$\left|E_P\right| \sim C(\theta)\left|\int_0^{2\pi}\int_0^{a} G(\rho)\exp\left[\frac{-jk\rho^2}{2R}\right]\exp[jk\rho\sin(\theta)\cos(\phi-\phi')]\rho d\rho d\phi'\right| \qquad (18)$$

where

$$C(\theta) = [1+\cos(\theta)]/4\pi R$$

With ϕ arbitrarily set to zero since the radiation field is circularly symmetric, the integration with respect to ϕ' can be performed through use of the identity (Stutzman and Thiele, 1981:416,566)

$$J_0(x) = (1/2\pi)\int_0^{2\pi}\exp[jx\cos(\alpha)]\ d\alpha \qquad (19)$$

so that the relative field strength is

$$|E_P| \sim D(\theta) \left| \int_0^a G(\rho) \exp\left[\frac{-jk\rho^2}{2R}\right] J_0[k\rho\sin(\theta)]\,\rho\,d\rho \right| \qquad (20)$$

where

$$D(\theta) = [1+\cos(\theta)]/2R$$

If the substitutions $x = \rho/a$, $\Delta = R/(2D^2/\lambda)$, and $u = ka\sin(\theta)$ are made, and the constants appearing outside the integral are dropped then

$$|E_P| \sim \frac{1 + \cos(\theta)}{\Delta} \left| \int_0^1 G(x)\, J_0(ux) \exp\left[\frac{-jkx^2}{8\Delta}\right] x\,dx \right| \qquad (15)$$

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FRESNEL REGION OF..(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI..   J E MIZE
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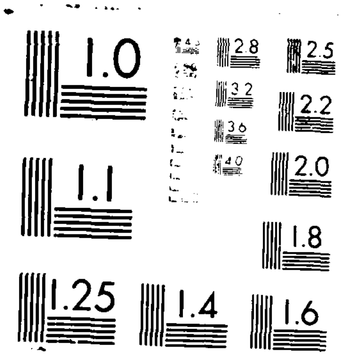
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Thesis Chairman: Andrew J. Terzuoli, PhD
Assistant Professor of Electrical Engineering

Approved for

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19. A computer program is presented which calculates power density in the Fresnel region of circular parabolic reflector antennas. The aperture illumination model is the one-parameter circular distribution developed by Hansen. The program is applicable to the analysis of electrically large, center-fed (or Cassegrain) paraboloids with linearly polarized feeds.

The scalar Kirchoff diffraction integral is solved numerically by Romberg integration for points both on and perpendicular to the antenna boresight. Axial results correspond with those published by Hansen. Off-axis results cannot be directly compared to any others obtained with this illumination model, but they are consistent with what is expected in the Fresnel region where a quadratic must be added to the linear phase term of the integral expression. Graphical results are presented for uniform illumination and for cases where the first sidelobe ratio is 20, 25, 30, and 35 dB.

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